Performance Analysis of Intelligent Reflecting Surface Selection for Orthogonal and Non-Orthogonal Multiple Access

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***Abstract*—Intelligent reflecting surface (IRS) can play a major role in relaying data in 6G networks. This paper studies the system performance of IRS selection (IRS-S) for both non-orthogonal multiple access (NOMA) and orthogonal multiple access (OMA). We first present two selection schemes in NOMA scenario for IRS relays, namely, the two-stage and the max-min IRS-S strategy. Then, we drive a closed-form expression for the system outage probability for the two-stage IRS-S scheme and the OMA scenario. In addition, simulation results are provided to validate the analytically driven expressions of the outage probability for the proposed schemes. The results confirm that the two-stage IRS-S strategy has superior performance compared to all selection schemes in both NOMA and OMA setups.**

I. INTRODUCTION

The emerging technology of intelligent reflecting surfaces (IRSs) has become a promising technique to extend coverage and enhance the capacity of the upcoming 6G networks [1]. The IRS is composed of a large number of low-cost, reconfigurable, and energy-efficient passive elements, where each particle of the surface can manipulate the electromagnetic wave with a certain phase shift and amplitude [2]. The IRS can be incorporated as relays to enhance the link quality and greatly improve the coverage [3]. Unlike conventional relaying, the IRS does not suffer from the high cost related to power consumption, latency, and dedicated processing at the relaying unit.

Another key technology in the multiple access domain is the non-orthogonal multiple access (NOMA), where several users (2 or more) are grouped to share the same resource block, either in time, frequency, code, or space [4]. The most widely known NOMA structure in the downlink is the power domain NOMA, where grouped users are discriminated by different power levels [4], which is the adopted NOMA system in this article and will thereafter be referred to as NOMA. In literature, several grouping criteria of users are discussed. In [4] and [5], users are grouped according to their channel conditions. While in [6], users are grouped based on the quality of service requirements. The basic principle is that one user is allocated more power and decodes its signal with interference from the second user, while the latter, that is assigned less power, applies successive interference cancellation (SIC) and is able to perfectly decode the signal of the first user i.e., removing the inter-user interference.

In order to maximize each NOMA users' rates, the IRS is integrated to support communication. Here, the superimposed signal is considered to be transmitted from the base station via one chosen out of many deployed IRSs in the system.

Most previous works have been devoted to IRS performance analysis considering a single IRS setup implemented in different wireless networks, where a performance comparison of only one IRS with decode-and-forward (DF) relay in [3] and another one with amplify-and-forward (AF) relay in [7] are considered. The comparison results showed that the IRS with a reasonable size and a certain number of reflecting elements outperforms the two types of relaying systems.

Nevertheless, several works have focused on the selection policies for multiple IRSs setup. In [8], a one-to-one stable matching algorithm is proposed. While in [9], a distance based user-IRS association is performed. In both cases, each IRS can be assigned to at most one user. The authors in [10] provided a performance comparison of the selection policy between DF relays and IRSs based on the maximization of the signal-to-noise ratio (SNR) for the uplink, in which the selection combining (SC) scheme and tools from stochastic geometry are utilized. However, in all the mentioned IRS selection works, only a single user is to be served by one IRS. Hence, performance evaluation of several relay selection strategies for cooperative NOMA, in the downlink, is needed to be considered for IRS selection, which is the main motivation behind our work.

One important related work that considers two users NOMA is presented in [11]. However, unlike this work that presents the outage probability of conventional relaying selection schemes, the presented work in this paper considers IRS relaying, which yields a different mathematical model due to the multiplication of the channels connecting the IRS with the base station and user. We derive closed-form expressions of the outage probability for both the two-stage IRS selection strategy and the OMA scenario. Moreover, simulations are carried out to corroborate the analysis, and to make a performance comparison between the proposed schemes.

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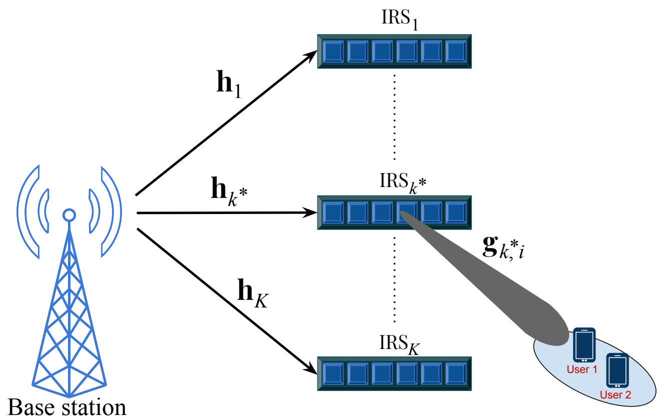

Figure 1. Multiple-element IRS-aided NOMA network selection model.

The rest of the paper is organized as follows: In section II, the system model is described. Section III presents the channel analysis and introduces the selection strategies for the IRS. In section IV, theoretical performance for the two-stage IRS selection and the OMA scenario in terms of outage probability is analyzed. In section V, numerical results are provided for verifying our analysis and comparing the performance of IRS selection strategies for both NOMA and OMA with different setups. Finally, our conclusions are drawn in section VI.

II. System Model and Setup

In this setup, we consider an IRS aided network as depicted in Fig. 1, where a base station (BS) that intends to communicate with two NOMA users, $i = 1, 2$, using the assistance of IRS k, where $k = \{1, \ldots, K\}$, with M reflecting elements for $m = \{1, ..., M\}$. Then, the IRS beamforms the signal to serve a single antenna user i. Due to physical obstacles that block the channel or very high degradation of the direct link between the BS and the users, the communication process is feasible only through the cooperating IRSs. The channel vector between the BS and the k^{th} IRS is denoted by $\mathbf{h}_k \in \mathbb{C}^M$ where $\mathbf{h}_k^{\mathrm{T}} = [h_{k,1}, ..., h_{k,m}, ..., h_{k,M}]^{\mathrm{T}}$ and the channel vector between the IRS and user i is denoted by $\mathbf{g}_{k,i} \in \mathbb{C}^M$ where $\mathbf{g}_{k,i} = [g_{k,1,i}, ..., g_{k,m,i}, ..., g_{k,M,i}]$. We define $\eta_{k,m} = |h_{k,m}|$ and $\upsilon_{k,m,i} = |g_{k,m,i}|$ follow complex Gaussian distribution with zero mean and unit variance; $\eta_m \sim \mathcal{CN}(0, 1)$ and $\upsilon_{k,m,i} \sim \mathcal{CN}(0, 1)$. Moreover, w_i denotes the additive white Gaussian noise at user i with zero mean and variance σ_i^2; $w_i \sim \mathcal{CN}(0, \sigma_i^2)$. We assume that no signal coupling in the reflection by neighboring IRS elements (all IRS elements reflect the incident signals independently), and that the channel state information (CSI) is known to the transmitter side.

For the proposed NOMA system, we assume that users are grouped by their quality of service (QoS) requirements, not by their channel conditions. We follow the same quality of service allocation in [11], where user 1 is assumed to be served for short-packet transmission, i.e., a reliable and quick connection with a low data rate. On the other hand, user 2 is performing trivial tasks that need a higher rate than user 1 and to be served opportunistically. Thus, according to NOMA power allocation coefficient policy, more power will be assigned to user 1. Consequently, interference at NOMA user 2 can be canceled by applying SIC technique.

The BS will transmit the two signals as a superimposed combination, $(\alpha_1 s_1 + \alpha_2 s_2)$, where s_i and α_i denote the i^{th} user signal and the power allocation coefficient assigned to this user, respectively. According to the NOMA scheme, we adopt the relationship $\alpha_1 > \alpha_2$ in order to meet the prioritized user quality of service requirements and $\alpha_1^2 + \alpha_2^2 = 1$ must be satisfied. Then, the IRS reflects the superposition of all incident signals since it is composed of M reflecting elements, where each element has a smaller size than the wavelength, thus it scatters the incoming signal with approximately constant gain in all directions of interest, hence, the properties of the k^{th} IRS can be fully represented by [12]

$$\mathbf{\Theta}_k = \mathrm{diag}\left(\beta_{k,1}e^{j\theta_{k,1}}, ..., \beta_{k,m}e^{j\theta_{k,m}}, ..., \beta_{k,M}e^{j\theta_{k,M}}\right), \quad (1)$$

where diag(.) represents the diagonal matrix of size $M \times M$ and $\beta_{k,m} \in (0, 1]$ is the amplitude-reflection coefficient. Without loss of generality, we assume that $\beta_{k,m} = \beta \; \forall k$ and is equal to 1 for maximal reflection, while $\theta_{k,m} \in [0, 2\pi)$ is the phase-shift variable of the m^{th} element that can be adjusted by the IRS. In our system, we assume user 1 who requires a higher quality of service than user 2, is our prioritized one in which the IRS is focused on providing maximum channel gain. Therefore, the main beam direction of the IRS, is designed towards user 1.

Assuming only first-order reflection from any selected IRS, the received signal from the BS through the k^{th} IRS to the two NOMA users is given by

$$r_{k,i} = \mathbf{h}_k^{\mathrm{T}} \mathbf{\Theta}_k \mathbf{g}_{k,i} \left(\alpha_1 s_1 + \alpha_2 s_2\right) + w_i. \quad (2)$$

For simplicity, we assume $w_i = w$. Then, user 1 treats the signal from user 2 as interference, and the signal-to-interference-plus-noise ratio (SINR) is given by [13]

$$\mathrm{SINR}_1 = \frac{\left|\mathbf{h}_k^{\mathrm{T}} \mathbf{\Theta}_k \mathbf{g}_{k,1}\right|^2 \alpha_1^2}{\left|\mathbf{h}_k^{\mathrm{T}} \mathbf{\Theta}_k \mathbf{g}_{k,1}\right|^2 \alpha_2^2 + \frac{1}{\rho}}. \quad (3)$$

where $\rho = P_t/\sigma_w^2$ represents the transmit SNR of the BS. However, since the phase shifts are designed for the prioritized user 1, user 2 detects its signal with SINR given by [14]

$$\mathrm{SINR}_2 = \frac{\left|\mathbf{h}_k^{\mathrm{T}} \mathbf{g}_{k,2}\right|^2 G_M\left(\theta_k\right) \alpha_1^2}{\left|\mathbf{h}_k^{\mathrm{T}} \mathbf{g}_{k,2}\right|^2 G_M\left(\theta_k\right) \alpha_2^2 + \frac{1}{\rho}}, \quad (4)$$

where $G_M(\theta_k)$ denotes the normalized Fejèr Kernel function with parameter M and period 2, hence, θ_k follows uniform distribution over $[-1, 1]$ [15]. Since users are paired according to QoS requirements, not by their channel conditions, let us assume that the paired NOMA users are served with the same beam. Moreover, to simplify the analysis similar to [13], we assume that both users share the same channel vector. Hence, SINR_2 can be written as

$$\mathrm{SINR}_2 = \frac{\left|\mathbf{h}_k^{\mathrm{T}}\mathbf{\Theta}_k\mathbf{g}_{k,2}\right|^2\alpha_2^2}{\left|\mathbf{h}_k^{\mathrm{T}}\mathbf{\Theta}_k\mathbf{g}_{k,2}\right|^2\alpha_1^2 + \frac{1}{\rho}}. \tag{5}$$

Thus, user 2 performs SIC, and the received SINR_2 becomes noise-limited, thereby, SNR_2 is equal to

$$\mathrm{SNR}_2 = \rho\left|\mathbf{h}_k^{\mathrm{T}}\mathbf{\Theta}_k\mathbf{g}_{k,2}\right|^2\alpha_2^2. \tag{6}$$

III. CHANNEL ANALYSIS AND SELECTION STRATEGIES

A. Study of the IRS channel analysis

We aim to provide a reliable communication between the BS and both users assisted by the k^{th} IRS, specifically, to optimize $\left|\mathbf{h}_k^{\mathrm{T}}\mathbf{\Theta}_k\mathbf{g}_{k,i}\right| = \left|\sum_{m=1}^{M} h_{k,m}g_{k,m,i}e^{j\theta_{k,m}}\right|$ by eliminating the channel phases. This can be achieved by intelligently adjusting the phase-shift variable $\theta_{k,m}$ for each element within the k^{th} IRS [16], i.e., the phases of all $e^{j\theta_{k,m}}$ are set to be the same. Thus the resultant solution is given by $\theta_{k,m} = \widetilde{\theta} - \arg(h_{k,m}g_{k,m,i})$ [17] where $\widetilde{\theta}$ is a constant ranging $\in [0, 2\pi)$. After adopting the optimal $\theta_{k,m}$, we have

$$\begin{aligned}\left|\mathbf{h}_k^{\mathrm{T}}\mathbf{\Theta}_k\mathbf{g}_{k,i}\right|^2 &= \left|\sum_{m=1}^{M} h_{k,m}g_{k,m,i}\right|^2 \\ &= \left(\sum_{m=1}^{M} |\eta_{k,m}||\upsilon_{k,m,i}|\right)^2 \\ &= \left(\sum_{m=1}^{M} X_{k,m,i}\right)^2 = Y_{k,i}^2. \end{aligned} \tag{7}$$

Based on insights from [18], the cumulative distribution function (CDF) of $Y_{k,i}^2$ can be approximated with a Gamma distribution and is equal to

$$F_{Y_{k,i}^2}(y) = \frac{1}{\Gamma(\kappa)}\gamma\left(\kappa, \frac{\sqrt{y}}{\vartheta}\right), \tag{8}$$

where $\kappa = \frac{M\pi^2}{16-\pi_2}$ and $\vartheta = \frac{16-\pi^2}{4\pi}$ denote the shape and scale parameter of the Gamma distribution, respectively. $\Gamma(.)$ is the Gamma function, and $\gamma(.,.)$ is the lower incomplete Gamma function.

B. IRS Selection Strategies

In this section, we propose two selection schemes to study.

1) Two-stage IRS selection: This hierarchical strategy is done in two stages. The first stage is to built a subset of IRSs by focusing on correctly decoding user 1's signal. Consequently, the first stage constraint to construct this subset can be formulated as

$$\mathcal{G} = \{k : 1 \leq k \leq K, Y_{k,i}^2 \geq \psi_1\}, \tag{9}$$

where $\psi_1 = \frac{\frac{\epsilon}{\rho}}{\alpha_1^2 - \epsilon\alpha_2^2}$ and $\epsilon = 2^{R_1} - 1$. We define $|\mathcal{G}|$ as the cardinality of $\mathcal{G}$. It is assumed that $\alpha_1^2 - \epsilon\alpha_2^2 > 0$, otherwise the first stage condition in (9), can not be satisfied. Thus, both users that are served by an IRS in $\mathcal{G}$ are capable of decoding s_1 successfully. Among the group of IRSs in $\mathcal{G}$, the second stage is to select an IRS that maximize the rate for user 2, i.e.,

$$k* = \arg_k \max\{Y_{k,2}^2, k \in \mathcal{G}\}. \tag{10}$$

2) Max-min IRS selection: The selection criterion for this strategy can be obtained as follows

$$\max\left\{\min\{Y_{k,1}^2, Y_{k,2}^2\}, k \in \mathcal{G}\right\}, \tag{11}$$

in which the IRS whose worse channel, $\min\{Y_{k,1}^2, Y_{k,2}^2\}$ is the best, is selected.

IV. PERFORMANCE ANALYSIS

In this section, the outage probability achieved by the two-stage IRS selection strategy is described. We consider the overall outage probability as the sum of two outage events. Each one represents the outage probability of each stage. Thus, the outage probability of the two-stage assisted-NOMA strategy can be written as

$$\mathrm{P}(\mathcal{O}_N) = \mathrm{P}(\mathcal{O}_{N_1}) + \mathrm{P}(\mathcal{O}_{N_2}), \tag{12}$$

where $\mathrm{P}(\mathcal{O}_{N_1})$ denotes the outage probability of the first stage, i.e., both users can not decode s_1 successfully $\forall k \in \{1, \ldots, K\}$, and $\mathrm{P}(\mathcal{O}_{N_2})$ denotes the outage probability of the second stage only given that the first stage is already passed, i.e., s_2 can not be decoded correctly by user 2, while s_1 is decoded successfully by the two users. Thus, the term $\mathrm{P}(\mathcal{O}_{N_1})$ can be written as follows

$$\mathrm{P}(\mathcal{O}_{N_1}) = \left[\frac{2\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}{\Gamma(\kappa)}\right]^K \tag{13}$$

Proof: Please refer to Appendix A. ■

To evaluate $\mathrm{P}(\mathcal{O}_{N_2})$, we assume that a subset of IRSs are formed from the first phase, i.e., $|\mathcal{G}| > 0$. Hence, according to the aim of the second stage, which is to select $Y_{k,2}^2$ that maximizes user 2's rate, we define

$$z_{k^*} = \max\{Y_{k,2}^2, \forall k \in \mathcal{G}\}. \tag{14}$$

Thus, the probability $\mathrm{P}(\mathcal{O}_{N_2})$ can now be expressed as follows

$$\mathrm{P}(\mathcal{O}_{N_2}) = \mathrm{P}(z_{k^*} < \psi_2, |\mathcal{G}| > 0), \tag{15}$$

where $\psi_2 = \frac{2^{R_2}-1}{\rho\alpha_2^2}$. The above probability can be written as

$$\begin{aligned}\mathrm{P}(\mathcal{O}_{N_2}) &= \sum_{n=1}^{K} \mathrm{P}(z_{k^*} < \psi_2, |\mathcal{G}| = n) \\ &= \sum_{n=1}^{K} \mathrm{P}(z_{k^*} < \psi_2 ||\mathcal{G}| = n)\mathrm{P}(|\mathcal{G}| = n) \\ &= \sum_{n=1}^{K} (F(\psi_2))^n \mathrm{P}(|\mathcal{G}| = n), \end{aligned} \tag{16}$$

where n is the number of IRSs satisfied the constraint in (9).

The CDF of ψ_2 can be written as

$$F(\psi_2) = \frac{\frac{1}{\Gamma(\kappa)}\gamma\left(\kappa, \frac{\sqrt{\psi_2}}{\vartheta}\right) - \frac{1}{\Gamma(\kappa)}\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}{1 - \frac{1}{\Gamma(\kappa)}\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}. \quad (17)$$

Proof: Please refer to Appendix B. ■

By invoking the binomial theorem, the probability that $|\mathcal{G}|$ have n IRSs can be calculated as

$$\begin{aligned}\mathrm{P}(|\mathcal{G}| = n) =& \binom{K}{n}\prod_{k=1}^{K-n}[1 - \mathrm{P}(Y^2_{\pi(k),1} > \psi_1)\\ &\times \mathrm{P}(Y^2_{\pi(k),2} > \psi_1)]\\ &\times \prod_{k=K-n+1}^{K}[\mathrm{P}(Y^2_{\pi(k),1} > \psi_1)\\ &\times \mathrm{P}(Y^2_{\pi(k),2} > \psi_1)],\end{aligned} \quad (18)$$

where $\pi(.)$ denotes possible random permutations of the IRSs. By exploiting the CDF obtained in Appendix A, the resultant expression of the above probability can be written as follows:

$$\mathrm{P}(|\mathcal{G}| = n) = \binom{K}{n}\left[\frac{2\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}{\Gamma(\kappa)}\right]^l\left[1 - \frac{2\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}{\Gamma(\kappa)}\right]^n, \quad (19)$$

where $l = K - n$. By substituting (17), (19) into (16), the overall outage probability for the two-stage IRS selection scheme can be given as

$$\begin{aligned}\mathrm{P}(\mathcal{O}_N) =& \sum_{n=0}^{K}\binom{K}{n}\left(F(\psi_2)\right)^n\left[\frac{2\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}{\Gamma(\kappa)}\right]^l\\ &\left[\frac{\Gamma(\kappa) - 2\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}{\Gamma(\kappa)}\right]^n.\end{aligned} \quad (20)$$

In order to gain more insights on the proposed IRS selection strategies for NOMA networks, the outage probability of the IRS selection for the paired users is also analyzed in the OMA scenario. The adopted OMA scheme in this article is time division multiple access (TDMA), where both users are supported with 2 identical time slots. In each time slot, the IRS provides access only for one of the users [13]. Thus, the achieved rate for each user individually can be expressed as

$$R_{i,O} = \frac{1}{2}\log_2(1 + \mathrm{SNR}_{i,O}) \quad (21)$$

Hence, the SNR of user i can be given as

$$\begin{aligned}\mathrm{SNR}_{i,O} =& \rho|\mathbf{h}_k^{\mathrm{T}}\mathbf{\Theta}_k\mathbf{g}_{k,i}|^2\\ =& \rho Y^2_{k,i}\end{aligned} \quad (22)$$

The selection criterion in the OMA scenario is defined as

$$x_{k^*,i} = \max\{Y^2_{k,i}, \forall k \in \{1, \dots, K\}\}. \quad (23)$$

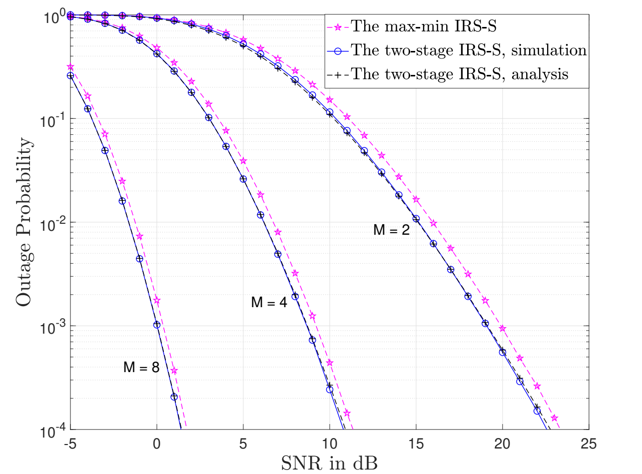

Figure 2. The outage performance of IRS-S strategies for NOMA setup with $R_1 = 0.5$ bit per channel use (BPCU), $R_2 = 2$ BPCU, $\alpha_1 = 3/4$, $\alpha_2 = 1/4$, $K = 2$ and different number of reflecting element.

which is to select an IRS with the highest channel gain for each user individually. Consequently, the outage probability for each user in OMA TDMA system can be given as

$$\begin{aligned}\mathrm{P}(\mathcal{O}_{O,i}) =& \prod_{k=1}^{K}\mathrm{P}(Y^2_{k,i} < \zeta_i)\\ =& \left(F(\zeta_i)\right)^K\end{aligned} \quad (24)$$

where $\zeta_i = \frac{2^{2R_i} - 1}{\rho}$. With the CDF obtained in (8), the outage probability can be written as

$$\mathrm{P}(\mathcal{O}_{O,i}) = \left[\frac{1}{\Gamma(\kappa)}\gamma\left(\kappa, \frac{\sqrt{\zeta_i}}{\vartheta}\right)\right]^K. \quad (25)$$

V. NUMERICAL RESULT

In this section, we provide theoretical and computer simulation results for the IRS with multiple-element aided-NOMA and OMA selection strategies.

Figure 2 illustrates the outage performance of NOMA IRS Selection for both the two-stage and the max-min schemes. Generally, increasing the number of elements of the IRS can efficiently reduce the outage. Moreover, the analytical and simulation curves match perfectly with each other.

Figure 3 compares the performance of IRS-S aided NOMA for symmetrical setup (both NOMA users have similar rate and channel gain i.e., $\psi_1 = \psi_2$), where the achieved outage rate of the two-stage scheme is the same as that of the max-min IRS-S strategy.

Proof: Please refer to Appendix C. ■

Figure 4 demonstrates the performance of IRS-S OMA scheme for TDMA with two identical time slots serving the two users, where a significant performance gain in terms of outage probability is achieved when increasing the number of reflecting elements.

In Figure 5, a comparison between NOMA selection strategies; the two-stage and the max-min schemes with the OMA

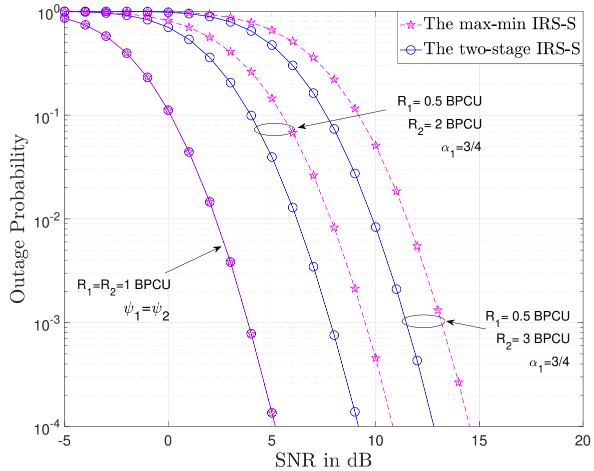

Figure 3. Comparison between the max-min and the two-stage selection schemes for different setups with $M = 2$ and $K = 10$.

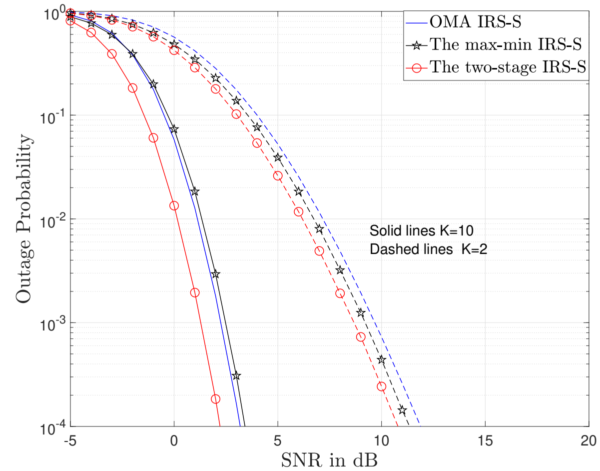

Figure 5. Comparison between IRS-S strategies for NOMA and OMA with $R_1 = 0.5$ BPCU, $R_2 = 2$ BPCU, with $M = 4$, $\alpha_1 = 3/4$, $\alpha_2 = 1/4$ and different number of IRSs.

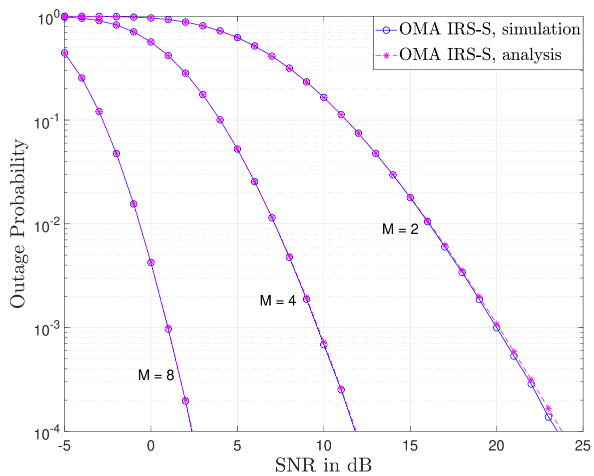

Figure 4. The outage performance of IRS-S strategy for OMA, $R_1 = 0.5$ BPCU, $R_2 = 2$ BPCU, $K = 2$ and different number of reflecting elements

selection strategy is demonstrated. As shown from the figure, the results confirm that the two-stage IRS-S strategy has superior performance compared to the the max-min IRS-S and OMA scheme. Moreover, the performance gab increases when deploying more IRSs in the system.

VI. Conclusions and Future Works

In this paper, IRS selection strategies for NOMA and OMA systems have been proposed for downlink transmission, where two main selection strategies for the IRS have been proposed; the two-stage and the max-min schemes. A closed-form expression for the outage probability achieved by the two-stage strategy and the OMA scheme has been derived. In general, the two-stage IRS Selection scheme achieves minimal outage probability. However, for symmetrical setup, the proposed NOMA schemes achieve the same outage performance. Furthermore, the outage performance improves as the IRSs are densely deployed or equipped with more reflecting elements.

For future work, the system throughput should be considered for different cases. Furthermore, it will be useful to investigate the impact of pathloss and shadowing in the system.

Appendix A

Derivation of $\mathrm{P}(\mathcal{O}_{N_1})$ in (13)

Based on the first stage condition in (9), we define the probability of the event that both NOMA users decode s_1 successfully for any $k \in \{1, \dots, K\}$ as

$$\mathrm{P}_{s_1} = \mathrm{P}(Y_{k,1}^2 > \psi_1)\mathrm{P}(Y_{k,2}^2 > \psi_1). \tag{26}$$

Therefore, the complementary event of the above probability, $\forall k \in \{1, \dots, K\}$, is $\mathrm{P}(\mathcal{O}_{N_1})$, which implies that $|\mathcal{G}| = 0$. Thus, the outage probability of the first stage can be given as

$$\begin{aligned}\mathrm{P}(\mathcal{O}_{N_1}) &= \prod_{k=1}^{K}[1 - \mathrm{P}(Y_{k,1}^2 > \psi_1)\mathrm{P}(Y_{k,2}^2 > \psi_1)] \\ &= \prod_{k=1}^{K}[1 - \bar{F}_{Y_{k,1}^2}(\psi_1)\bar{F}_{Y_{k,2}^2}(\psi_1)],\end{aligned} \tag{27}$$

where $\bar{F}_{Y_{k,i}^2}(.)$ is the complementary CDF (CCDF) defined as $\bar{F}_{Y_{k,i}^2}(.) = 1 - F_{Y_{k,i}^2}(.)$. By utilizing the CDF in (8), $\mathrm{P}(\mathcal{O}_{N_1})$ can be expressed as

$$\begin{aligned}\mathrm{P}(\mathcal{O}_{N_1}) &= \left[1 - \left(1 - \frac{\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}{\Gamma(\kappa)}\right)^2\right]^K \\ &= \left[\frac{2\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}{\Gamma(\kappa)} - \left(\frac{\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}{\Gamma(\kappa)}\right)^2\right]^K,\end{aligned} \tag{28}$$

since the contribution of the second term from the above equation is negligible, hence, it can be ignored, and $\mathrm{P}(\mathcal{O}_{N_1})$ can be reduced to (13).

APPENDIX B
DERIVATION OF THE CDF OF ψ_2 IN (18)

For an IRS randomly selected from $\mathcal{G}$, denoted by IRS k, the CDF of z_{k^*} can be founded as follows

$$\begin{aligned} F(z) =& \mathrm{P}\big(Y_{k,2}^2 < z \big| k \in \mathcal{G}, |\mathcal{G}| \neq 0\big) \\ =& \mathrm{P}\big(Y_{k,2}^2 < z \big| Y_{k,2}^2 > \psi_1\big). \end{aligned} \tag{29}$$

Thus, the above conditional probability of the CDF can be expressed as follows

$$F(z) = \frac{\mathrm{P}\big(Y_{k,2}^2 < z, Y_{k,2}^2 > \psi_1\big)}{\mathrm{P}(Y_{k,2}^2 > \psi_1)}, \tag{30}$$

which is the same as

$$\begin{aligned} F(z) &= \frac{\mathrm{P}(\psi_1 < Y_{k,2}^2 < z)}{\mathrm{P}(Y_{k,2}^2 > \psi_1)} \\ &= \frac{F_{Y_{k,2}^2}(z) - F_{Y_{k,2}^2}(\psi_1)}{\bar{F}_{Y_{k,2}^2}(\psi_1)}. \end{aligned} \tag{31}$$

Hence, the CDF can be written as follows

$$F(z) = \frac{\frac{1}{\Gamma(\kappa)}\gamma\left(\kappa, \frac{\sqrt{z}}{\vartheta}\right) - \frac{1}{\Gamma(\kappa)}\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}{1 - \frac{1}{\Gamma(\kappa)}\gamma\left(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\right)}. \tag{32}$$

It should be noted that $Y_{k,2}^2$ should be larger than ψ_1, which is due to the fact that IRS k belongs to $\mathcal{G}$, consequently, we can uphold the following for $F(z)$ in (32)

$$F(\psi_1) = 0 \quad \text{and} \quad F(\infty) = 1. \tag{33}$$

Therefore, by substituting ψ_2 to (32), the CDF for ψ_2 can be derived as given in (18)

APPENDIX C
SYMMETRICAL SETUP OUTAGE PERFORMANCE

For the general case, the overall outage probability for the proposed two-stage IRS-S can be derived as follows

$$\begin{aligned} \mathrm{P}_u(\mathcal{O}) =& \mathrm{P}\big(Y_{k,1}^2 < \psi_1\big) + \mathrm{P}\big(Y_{k,2}^2 < \psi_1, Y_{k,1}^2 > \psi_1\big) \\ &+ \mathrm{P}\big(Y_{k,2}^2 < \psi_2, Y_{k,2}^2 > \psi_1, Y_{k,1}^2 > \psi_1\big). \end{aligned} \tag{34}$$

Then, when $\psi_1 = \psi_2$, we get

$$\mathrm{P}_u(\mathcal{O}) = \mathrm{P}\big(Y_{k,1}^2 < \psi_1\big) + \mathrm{P}\big(Y_{k,2}^2 < \psi_1, Y_{k,1}^2 > \psi_1\big). \tag{35}$$

Hence, the above equation can be written as

$$\mathrm{P}_u(\mathcal{O}) = \mathrm{P}(\min\{Y_{k,1}^2, Y_{k,2}^2\} < \psi_1), \tag{36}$$

which is exactly the same as the outage probability of the max-min IRS-S approach for symmetrical setup and is equal

$$\begin{aligned} \mathrm{P}_u(\mathcal{O}) =& \mathrm{P}(\min\{Y_{k,1}^2, Y_{k,2}^2\} < \psi_1, \forall k \in \{1, \dots, K\}\big) \\ =& \mathrm{P}(\min\{Y_{\pi(1),1}^2, Y_{\pi(1),2}^2\} < \psi_1\big)^K. \end{aligned} \tag{37}$$

By exploiting the CDF in (8), The final expression of the outage probability for symmetrical setup can be written as

$$\mathrm{P}_u(\mathcal{O}) = \left[\frac{2\gamma\big(\kappa, \frac{\sqrt{\psi_1}}{\vartheta}\big)}{\Gamma(\kappa)}\right]^K. \tag{38}$$

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