Optimizing the Array Factor of a Phased Array Antenna (PAA) using a genetic algorithm

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***Abstract*—Phased Array Antennas are among the most important 5G enabling technologies. Optimizing a Phased Array Antenna (PAA) is an elaborate task, especially for PAAs with increased number of elements. Traditional optimization methods that are either based on greedy, local optimization such as gradient methods or consist of random walk solution space searches, are far from ideal for solving the problem of arbitrary pattern synthesis in 1D and 2D antenna arrays, mainly due to the high dimensional, multimodal functional domains involved but also due to the inherent limitations of these approaches e.g., the majority of these methods assume the object function is continuous or even differentiable. On the other hand, natural optimization methods, e.g., genetic algorithms, simulated annealing and particle swarm optimization can be used to tackle this problem efficiently. In this study, a genetic algorithm was developed for optimizing a Planar (2D) Phased Array Antenna, using a custom objective function so that the Array Factor has specific characteristics, in terms of directivity, beamwidth and side-lobe level (SLL). The experimental results showed that the genetic algorithm can be used for optimizing a 2D PAA with very satisfactory results.**

***Index Terms*—component, formatting, style, styling, insert**

I. Introduction

The 5th Generation (5G) of cellular networks is more than an extension of the capabilities offered by the well-established 4G LTE, mainly dealing with new techniques and technologies and trying to meet the requirements of the state-of-the-art applications e.g., Virtual Reality, Augmented Reality, Autonomous Vehicles, Internet-of-Things or Industrial Automation, which are now on the verge of massive commercialization [1]. This brings new challenges to the design of 5G products requiring high throughput connectivity, low latency, and exceptional area coverage. At the same time, 5G promotes the introduction of millimeter wave (mmWave) frequencies with larger available spectrum [2], investigating several frequency bands beyond the 28 GHz [3], such as V- or W-band [4]. As networks and applications grow, the precise simulation of antennas is the key to the development of the next generation of broadband products.

Phased-array antennas have been used in a variety of applications for decades. The advent of 5G, rendered them more essential than ever in mobile wireless communications, as the frequencies have increased, allowing for smaller antenna arrays. 5G systems operating in the Frequency Range 2 (FR2) band (24.25 GHz to 52.6 GHz) make good use of phased array antennas featuring beamforming, beam steering and beam-combining, requiring careful optimizing of the PAAs characteristics.

Natural optimization algorithms, a broad class of optimization algorithms that mimic natural processes, have been employed in a series of research efforts for optimizing various design aspects of PAAs, since traditional optimization approaches, that are either based on greedy, local optimization methods such as gradient methods or consist of random walk solution space searches, are far from ideal for high dimensional problems and problems involving multimodal functional domains [5]. One of the most widely used natural optimization algorithms, is the genetic algorithm (GA). Introduced by Holland [6] and inspired from the principles of genetics and the notion of natural selection, it is an optimization and search technique that thrives where traditional optimization approaches fail, having the ability to escape from local minima and maxima [7] while also being able to run in parallel, thus significantly reducing execution time.

Genetic algorithms have been employed for optimizing various design aspects of array antennas of various geometries e.g., uniformly spaced linear arrays, non-uniformly spaced linear arrays, planar arrays and for various applications e.g., radio astronomy [8], satellite communications, mmWave MIMO systems etc. In 2007, Villegas et al. proposed using a GA to optimize the shaped beam coverage areas of planar 2D phased arrays, using a planar rectangular array of 10×10 elements as a case study [9]. In 2015, Rocca et al. proposed decomposing an array structure in subarrays with irregular polyomino tiles whose locations and orientations are optimized by means of a genetic algorithm-based approach, as a solution for designing phased arrays, generating low sidelobes and grating-lobes-free patterns over wide frequency bandwidths [10]. In [11] a genetic algorithm was used for synthesizing a two-way antenna array pattern of a radar, while in [12] a method for designing and calibrating a millimeter-wave (mm-wave) multiple-input multiple-output (MIMO) antenna module was developed. Authors in [13] used a genetic algorithm (GA) to synthesize and optimize the excitation weights of a planar array for satellite communications based on 2016 ITU Radio Regulations. Soltankarimi, F. and al. showed in [14], that the genetic algorithm provides efficient and optimum solutions among a pool of candidate solutions in order to achieve the desired array performance for the purposes of radio astronomy. Authors in [15] demonstrated the optimization of a linear antenna array by varying the spacing of the antenna elements as well as by adjusting the feed current amplitudes and in [16] the synthesis of a linear antenna array using genetic algorithm to reduce peak sidelobe level was examined and the thinning of array was achieved through iteratively applying genetic algorithm, while in [17], a GA was employed for generating the locations of the elements reducing the SLL of a linear array of length $L = 1,5\lambda$. In [18] a GA was employed for minimizing the total output power of a 1D Linear Array of 100 elements, in the direction of the interfering signals. A GA was employed in [19], for the design of a Linear Array Antenna with cosecant and flat-topped beam patterns. Table I is featuring a state-of-the-art of some of the most recent papers on antenna optimization parameters and a comparison to our work.

TABLE I
A SUMMARY OF SOME OF THE MOST RECENT WORKS COMPARED TO THIS WORK

Year	Dim	N_{elements}	Optimization Goal(s)	Tunable parameter	Ref
2005	1D/ 2D	100, 10×10	SLL Reduction	Amplitude & phase excitation	[14]
2008	1D	7	SLL Reduction	Amplitude & phase excitation	[19]
2009	1D	100	Minimization of total output power in the direction of the interfering signals	Amplitude & phase excitation	[18]
2015	2D	$M \times N$: M,N$\in\{32, 40, 48, 54, 62\}$	SLL Reduction, Grating lobes elimination	Subarrays' orientation & position	[10]
2015	1D	64, 128	SLL Reduction, Thinning	Amplitude & phase excitation	[16]
2018	1D/ 2D	8, 32 × 32	SLL Reduction	Elemets' combination	[11]
2018	1D	20	SLL Reduction	Elements' locations	[17]
2019	2D	16 × 16	Compliance with the 2016 ITU Mask	Amplitude and phase excitation	[13]
2020	2D	8 × 8	Far-field calibration	Amplitude & phase excitation	[12]
2022	2D	3 × 11	SLL Reduction, desired FNBW, desired directivity	Amplitude & phase excitation	our work

This work aims to contribute to the existing literature, by demonstrating the capability of genetic algorithms to solve the problem of arbitrary pattern synthesis in planar Phased Array Antennas by optimizing the amplitude and phase of the current of the elements of the array to achieve the desired beamwidth, directivity and SLL reduction. In our simulations, a planar Phased Array Antenna of dimensions 3×11, almost similar to the V-band Phased Antenna Array (PAA) prototype used in [20], was used as a case study and the proper fitness function was formulated. This function uses euclidean distances to quantify the fitness of the output. One can find similar efforts in the bibliography (Table I), however most of them are either examining 1D arrays or using only one optimization criterion. For example, in [14] the sidelobe level (SLL) of linear and planar phased array is optimized using a genetic algorithm (GA) but the beamwidth and the directivity are not taken into consideration. To the best of our knowledge, this is the first time that all criteria of practical interest, namely directivity, beamwidth and side-lobe level, are taken into consideration in the optimization of a PAA.

II. Methodology

A. Phased Array Antennas

Phased array antennas consist of multiple stationary antenna elements, which are fed coherently while variable phase and time-delay control is used at each element to scan a beam to given angles in space [21]. Time delay can also be emulated with a phase shift [22], which is common and practical in many implementations, as shown in Figure 1.

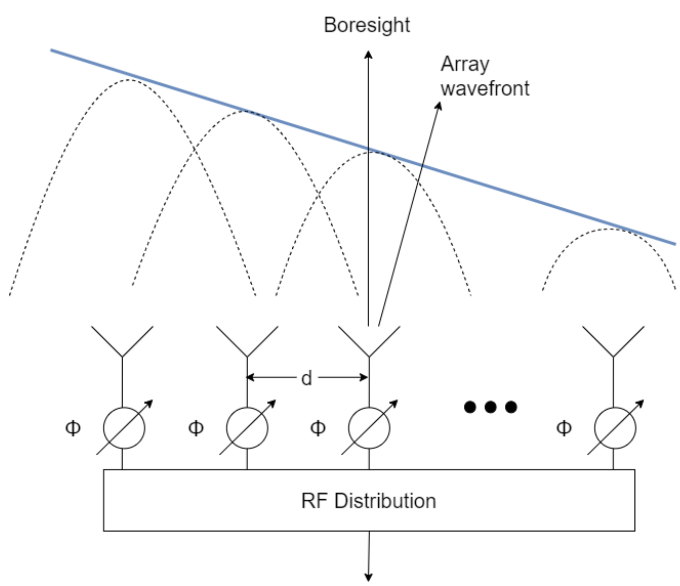

Fig. 1. Linear Phased Array Antenna, using phase shift control.

B. The Array Factor

The fitness function employed for phased array optimization is usually based on the array factor that can be formulated as a function of the relative position and amplitudes/phases of all the elements. The array factor AF is the complex-valued far-field radiation pattern obtained for an array of N isotropic radiators (elements) located at coordinates $\bar{r}_n$, as determined (in spherical coordinates) by:

$$AF(\hat{r}) = \sum_{n=1}^{N} \alpha_n e^{jk \cdot \hat{r}\bar{r}_n} \tag{1}$$

where α_n are the complex-valued excitation coefficients and $\hat{r}$ is the direction unit vector [21].

The array factor of a planar phased array antenna of dimensions $M \times N$, like the one that was employed as a case study for testing the proposed GA (Figure 2), is calculated as:

$$AF = \sum_{n=1}^{N} \sum_{n=1}^{M} \alpha_{mn} e^{j(m-1)\psi_x} \cdot e^{j(n-1)\psi_y} \tag{2}$$

where $\psi_x = kd_x \sin\theta \cos\phi + \beta_x$ and $\psi_y = kd_y \sin\theta \sin\phi + \beta_y$ are the wavefunctions corresponding to the x and y axis respectively, k is the wavenumber, d_x, d_y are the distances between the elements of the array across the axes x and y respectively and β_x,β_y are the phases between two successive elements across the axes x and y respectively.

C. Array Factor Characteristics

The Array Factor can be described through various quantifiable characteristics. For the purposes of this study, three of them were examined: beamwidth, directivity and side-lobe level (SLL). Each of these characteristics is of paramount importance when the optimization of a Phased Array Antenna is performed. Beamwidth provides a metric of angular resolution for antennas. Most commonly, beamwidth is defined by either the half-power beamwidth (HPBW) or the null-to-null spacing of the main lobe (FNBW), which is the case employed in this study. Antenna beamwidth is of utmost importance, since it determines the expected signal strength given the direction and radiation distance of an antenna. Directivity of an antenna is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.

$$D = \frac{P\,(\text{angle})}{P_{avg}} \tag{3}$$

When no direction is defined, directivity is determined by Equation (4).

$$D = \frac{P_{max}}{P_{avg}} \tag{4}$$

where P_{max} is the global maximum value of the Array Factor. Directivity is also a very important characteristic of the Array Factor of a PAA, since high directivity is very important for 5G communication. Last, but not least, side-lobe level (SLL) is the proportion of the field strength of the main lobe to the field strength of the side-lobes. High SLL leads to excessive sidelobe radiation, a phenomenon that wastes energy and may cause interference to other equipment or other signals.

D. The Problem Formulation

The planar Phased Array Antenna examined as a case study, is one of dimensions 3×11 and similar to the V-band Phased Antenna Array (PAA) prototype used in [20]. Such PAAs, operating on the V-band, offer the necessary bandwidth for mmWave applications while on the same time do not suffer from high losses. That's why they are among the most crucial 5G enabling technologies.

Let a planar Phased Array Antenna of $M \times N$ elements, where $M = 3$ and $N = 11$ as shown in Figure 2. Each element i, j has a tunable amplitude $A[i,j] \in \{0,1,2,3,4,5\}$ and a tunable phase $P[i,j] \in \{0, 90°, 180°, 270°\}$. It is worth mentioning that amplitude is not expressed in any predefined unit, but depends on the proprietary design of each antenna. The discrete values that the amplitude and the phase of each element were not chosen arbitrarily, but according to the tuning options also offered by the Phased Antenna Array (PAA) prototype used in [20]. Our aim is to find the variables:

$$A[i,j]\,,\ P[i,j]\ \forall i,j : i \in [1,M]\,\&j \in [1,N] \tag{5}$$

for which the Array Factor takes a form that satisfied specific user-defined criteria that have to do with directivity, beamwidth and the sidelobe level (SLL). In particular, the main lobe has to point towards a desired angle, the beamwidth has to have a preferred value and the field strength of the side lobes have to be adequately lower (defined in dB) compared to the field strength of the main lobe.

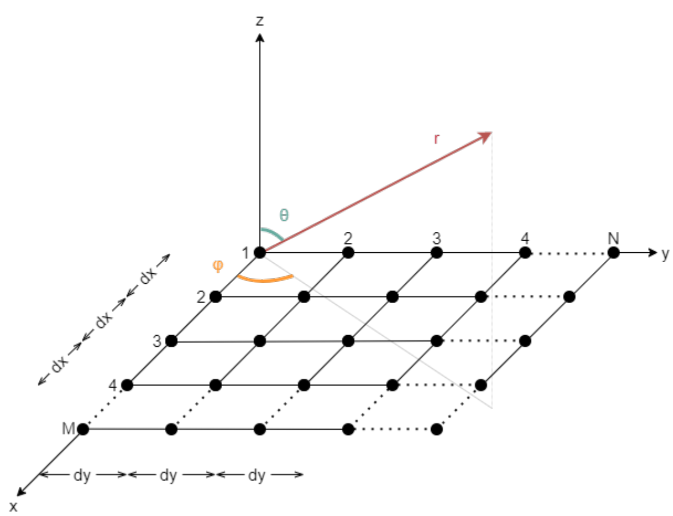

Fig. 2. A $M \times N$ 2D Planar Array

E. Genetic Algorithm workflow

The Genetic Algorithm (GA) optimization methodology is based on Darwinian rules of evolutionary dynamics, i.e., the stochastic operators of selection, crossover, and mutation. The GA begins, like any other optimization algorithm, by defining the optimization variables and the fitness function and ends by testing for convergence. The genetic algorithm workflow is shown as a flowchart in Figure 3.

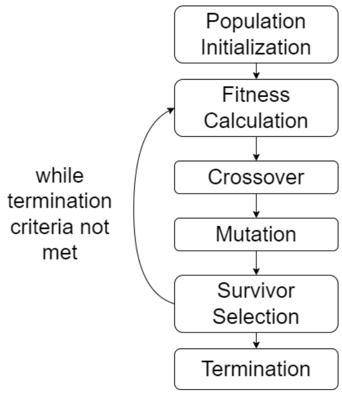

Fig. 3. Genetic Algorithm Flowchart

One can see in Figure 3 that the stochastic operators of crossover, mutation and selection are employed [23]–[25]. In this study, the termination criterion applied is the maximum number of iterations.

F. Fitness Function

The fitness function used, was developed, in an effort to find the optimal way to quantify the fitness of the output. It can be formulated as the sum of the three penalty terms, each corresponding to one of the three optimization requrements:

$$f_{\text{fitness}} = p_{\text{side-lobes}} + p_{\text{beamwidth}} + p_{\text{directivity}} \tag{6}$$

where the penalty term corresponding to the side-lobe level (SLL) is calculated as:

$$p_{\text{side-lobes}} = \frac{1}{N} \sum_{i}^{N} \left(AF\left[m\right] - AF\left[\ell_i\right] \right)^2 \tag{7}$$

the beamwidth is calculated as:

$$p_{\text{beamwidth}} = \left(FNBW_{AF} - FNBW_{\text{desired}} \right)^2 \tag{8}$$

and the directivity is calculated as:

$$p_{\text{directivity}} = \left(\theta_{\text{main lobe}} - \theta_{\text{desired}} \right)^2 \tag{9}$$

To ensure convergence of the GA, the optimization goals are set taking into account the physical properties of the array in terms of beamwidth, and side-lobe level. For the optimization algorithm to converge, an extra step had to be taken, guaranteeing that the main lobe is neither at the first nor at the last local maximum of the Array Factor, so that the main lobe will have at least one side-lobe on its left and at least one side lobe on its right. Towards that, one can also arbitrarily define the range of θ, so that the θ_{desired} is not close to the edges of the selected range. To achieve this the following algorithm was used for calculating the fitness function:

1) Find local maxima of the Array Factor. Let N be the number of the local maxima and:

$$\ell_1, \ell_2, \ldots, \ell_N \tag{10}$$

be the indices corresponding to the local maxima, indicated by the red dots in Figures 4 and 5.

2) Find the peak of the main lobe, as the maximum among the local maxima. Let m be the index of the peak corresponding to the main lobe. If $m = \ell_1$ or $m = \ell_N$:

$$f_{\text{fitness}} = 1000 \tag{11}$$

Else (if $m \neq \ell_1$ and $m \neq \ell_N$) the fitness function is calculated according to Equation (6).

III. Results

We performed the simulation for different values of the penalty parameter. In this work, the results of two runs of the algorithm are shown indicatively. In the first run, the desired FNBW was set equal to 30°, the directivity was set equal to 0° and the difference between the field strength of the main lobe and each of the side-lobes was set equal to 15 dB. The amplitudes and the phases suggested by the algorithm, were calculated as shown in Table II.

TABLE II
Amplitudes & Phases

A	4	5	4	3	1	5	5	0	3	4	0
A	3	2	0	2	0	2	1	5	1	4	3
A	4	5	4	2	4	0	0	2	5	4	0
P	270	270	0	0	90	270	0	N/A	0	0	N/A
P	270	0	N/A	90	N/A	0	0	90	180	90	90
P	270	270	0	270	0	N/A	N/A	270	0	0	N/A

It is obvious that the Array Factor satisfies the user-defined criteria. The FNBW achieved is equal to 32.94°, deviating only by 9.8 % from the desired FNBW (30°). In Figure 4 one can also see that the average difference between the field strength of the main lobe and the field strength of each of the rest local maxima of the Array Factor is equal to $14.5 \pm 1.15\,dB$, which is very close to the desired 15 dB. Last but not least the direction of the main lobe is exactly at 0°.

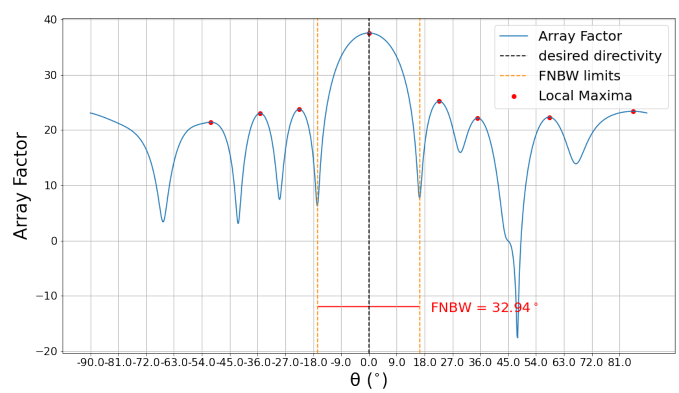

Fig. 4. The Array Factor for $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (FNBW $= 30°$, $\theta_{\text{desired}} = 0°$, SLL $= 15\,dB$)

In the second run, the desired FNBW was set equal to 30°, the desired directivity was set equal to 45° and the desired difference between the field strength of the main lobe and each of the side-lobes was set equal to 20 dB. The amplitudes and the phases suggested by the algorithm, were calculated as shown in the following two tables: each of the side-lobes was set equal to $20\,dB$.

TABLE III
AMPLITUDES & PHASES

A	1	1	4	1	3	3	4	3	0	2	0
	0	4	1	4	1	3	5	5	5	2	5
	2	0	0	1	2	0	0	4	2	5	4
P	90	90	270	270	90	0	270	180	N/A	270	N/A
	N/A	270	0	90	0	270	180	90	0	90	180
	270	N/A	N/A	0	0	N/A	N/A	0	180	270	90

It is obvious that the Array Factor satisfies the user-defined criteria. The FNBW achieved is equal to $29.7°$, deviating only by 1 % from the desired FNBW ($30°$). In Figure 5 one can also see that the average difference between the field strength of the main lobe and the field strength of each of the rest local maxima of the Array Factor is equal to $19.5 \pm 1.2\,dB$, which is very close to the desired $20\,dB$. Last but not least the direction of the main lobe is exactly at $45°$.

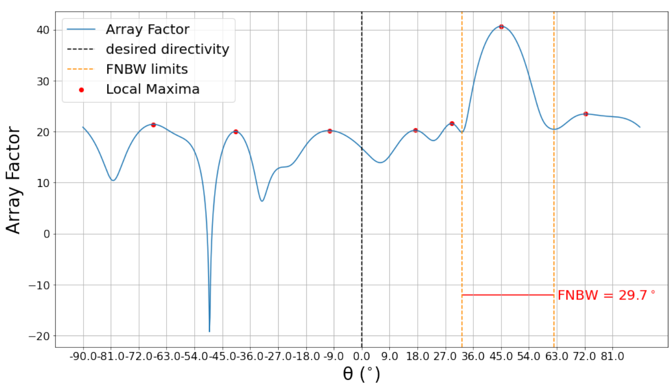

Fig. 5. The Array Factor for $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (FNBW $= 30°$, $\theta_{\text{desired}} = 45°$, SLL $= 20\,dB$)

IV. Conclusions

In this paper, a genetic algorithm was employed for optimizing a planar Phased Array Antenna of dimensions 3×11, so that its array factor has the desired characteristics. Through the optimization process, the fitness function was formulated using Euclidean distances to measure the fitness of the solution. It was proven, that one can use a genetic algorithm to find a combination of amplitudes and phases for each element of a Phased Array Antenna, so that the main lobe of the Array Factor has the desired directivity, the beamwidth and its field strength is adequately larger (i.e., larger by a user-defined amount) than the field strength of the side lobes. The optimization procedure required thousands of iterations and had a duration of a few hours.

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