Decentralized Oracles with Threshold Signatures : A Discrete Public Goods Game Model

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***Abstract*—The blockchain oracle problem is a central challenge in the integration of blockchain in decentralized systems. Ensuring that off-chain data fed into smart contracts is reliable is a problem, and relying on a single oracle introduces a single point of failure. To address this, several decentralized oracle designs have been proposed, including those based on threshold signature schemes. In such systems, a data feed is accepted only if a minimum number of oracles sign it. While this improves robustness, it introduces coordination issues: signing incurs a cost, and individual oracles may prefer to free-ride, expecting others to sign. In this work, we model oracle participation as a discrete public goods game and analyze the conditions under which signing is a rational strategy in equilibrium. We characterize the set of pure and symmetric mixed-strategy Nash equilibria and study how key system parameters, such as the number of oracles, the cost-to-reward ratio, and the signature threshold, affect participation incentives. Our results show that system parameters can give rise to multiple symmetric mixed-strategy equilibria, but that such equilibria disappear when the signing cost reaches as little as 27.5% of the reward.**

***Index Terms*—Blockchain, Decentralized Oracles, Game-Theory, Public Good Games.**

I. Introduction

Digital systems increasingly rely on decentralized trust. The automation of these systems is largely driven by smart contracts, which allow users to execute transactions and enforce agreements directly on the blockchain without a trusted third party [1]. Smart contracts are now a big component to various decentralized applications, including finance, insurance, and supply chain management. However, smart contracts cannot access external data on their own. To retrieve off-chain information such as asset prices, weather data, or sensor readings, they depend on oracles.

An oracle is an agent that delivers real-world data to a smart contract. If the data provided is incorrect or manipulated, the contract may execute an invalid outcome. This vulnerability is known as the blockchain oracle problem [2]. Relying on a single oracle introduces a single point of failure, undermining the reliability of decentralized systems. To address this issue, several decentralized oracle systems have been proposed [3]. These systems typically aggregate inputs from multiple oracles and accept a data feed only when a minimum number of them agree. One method to implement this is through threshold signatures, where at least t out of n oracles must sign a data feed for it to be accepted by the smart contract [4].

Threshold signature schemes offer strong security guarantees while reducing verification complexity, as only a single aggregate signature must be verified instead of individual ones. However, they introduce coordination challenges. Signing consumes oracle resources and incurs a cost. If only t signatures are required, oracles may choose not to participate and hope others will, thereby benefiting without contributing. This free-rider problem may prevent the system from reaching the required threshold, leading to failure.

In this paper, we analyze the strategic behavior of oracles in a threshold signature system. We model their interaction as a discrete public goods game, where the contribution is a partial signature on the data feed, and a reward is distributed only if the threshold of signers is met. Using game theory, we identify the conditions under which decentralized participation is a rational strategy. Our key contributions are summarized as follows:

- We model threshold signature-based decentralized oracles as a discrete public goods game.
- We analyze the pure and symmetric mixed-strategy Nash equilibria.
- We perform a numerical evaluation of the symmetric mixed-strategy equilibria, examining how they vary with the number of oracles, the cost-to-reward ratio, and the signing threshold.
- We identify system configurations where non-participation becomes a dominant strategy.

The remainder of this paper is organized as follows. Section II reviews related work on oracle systems and decentralized incentive mechanisms. Section III presents the game model, formalizing oracle participation as a discrete public goods game and analyzing both pure and mixed-strategy equilibria. Section IV provides a numerical evaluation of the equilibria under different system parameters. Section V concludes the paper.

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II. Related Work

Several works have tried to solve the blockchain oracle problem. One approach involves staking-based consensus mechanisms. The protocol in [5] introduces a decentralized oracle system where participants act as submitters, reporters, and certifiers, each staking assets to gain voting rights. Votes are weighted by the amount staked, and the system includes rewards for agreement and penalties for disagreement. The authors model the system as a voting game and show that, assuming an honest majority, truthful voting is a Nash equilibrium. They identify two main issues—lazy voting and deceitful voting—and analyze the level of accuracy required for voters to remain profitable. This model is extended in [6], where voters assign confidence scores instead of binary votes. The authors remove the role of certifiers and show that the new scheme still leads to honest behavior.

Another approach introduces reputation mechanisms. Deepthought [7] extends the model in [5] by combining both stake and reputation when weighting votes, with stake having a slightly higher influence. Voters gain or lose reputation based on whether their vote aligns with the majority. Rewards are assigned based on consistency with the voter's stated belief, rather than agreement alone. The system is designed for scenarios involving human-generated content, such as news, rather than sensor data. Another related work is BIVO [8], where the authors propose a combined voting mechanism that takes into account both the oracle's reputation and its vote. They also use a stochastic game framework to analyze repeated voting strategies in the presence of adversarial oracles.

Several market-ready platforms implement incentive-compatible mechanisms. Augur [9] operates as a decentralized prediction market where participants are rewarded or penalized through reputation tokens. Witnet [10] follows a similar model, with oracles earning or losing reputation based on data accuracy. Chainlink [11] provides a more flexible infrastructure, using decentralized aggregation, performance-based reputation scores, and hybrid smart contracts to enable secure off-chain computation.

More recent work focuses on robustness and scalability using cryptographic techniques. In [4], the authors introduce a mechanism combining multi-threshold aggregate signatures with a reputation layer. Only nodes meeting reputation requirements can generate valid signatures, reducing gas costs and improving security. However, scalability remains a challenge. In a similar work, [12] proposes a hybrid committee model for Internet-of-Things (IoT) data verification. Trusted execution environments are combined with threshold signature schemes to accelerate response times and filter unreliable input.

In this work, we focus on the class of decentralized oracle systems, where a minimum number of signatures is required to validate a data feed. We model oracle participation as a discrete public goods game and analyze the strategic behavior that arises when threshold signatures are used to aggregate attestations.

III. Game Model

A. Preliminaries and Assumptions

We consider a setting in which a deployed smart contract on a blockchain platform requires a trusted data feed, attested by decentralized oracles. Specifically, at least t out of n oracle nodes must attest on the data-feed for it to be accepted by the contract. We assume the oracles employ a threshold signature scheme. In such a scheme, at least t oracle nodes must partially sign a message for the resulting signature to be valid and verifiable under a shared public key. Once the threshold is reached, the smart contract verifies the aggregated signature.

Each oracle that receives a data message must choose between two actions: either sign the message and broadcast it to the other oracles, or simply drop it without taking further action. For simplicity, we focus on the decision to sign, which incurs a cost c. Choosing not to drop is cost-free. We assume all oracles take the action simultaneously.

A reward of value r is distributed uniformly among participating oracles if the number of signatures reaches or exceeds the threshold t. If fewer than t oracles sign, the data-feed is not accepted by the smart contract, and no reward is issued. An overview of the studied system is illustrated in Figure 1.

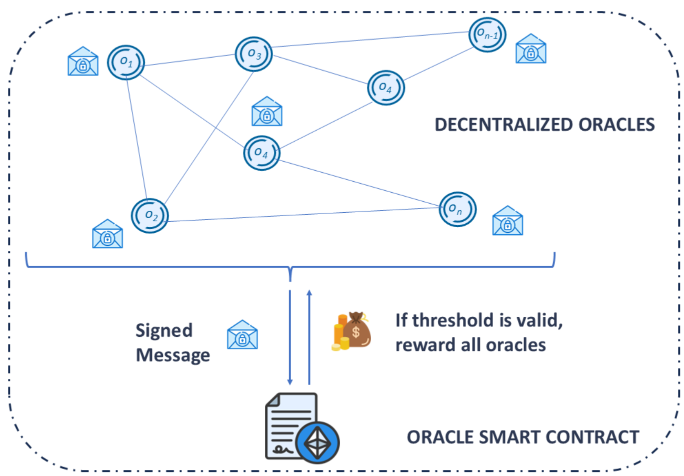

Fig. 1: Model Illustration

We move to formally model the strategic interactions between the signing oracles.

B. Threshold Signature Game

We model the threshold signature participation of decentralized oracles as an N-player strategic-form game. The game is defined by the tuple $\mathcal{G} = \langle \mathcal{P}, \mathcal{A}, (u_i)_{i \in \mathcal{P}} \rangle$.

The Players $\mathcal{P}$: The oracles are modeled as the set of players $\mathcal{P}$, where $\mathcal{P} = \{1, 2, 3, ..., N\}$ and $N \in \mathbb{N}$.

The Action Space $\mathcal{A}$: Each player receives a data feed message that needs to be signed by at least t oracles. Upon receiving the message, the player chooses one of two actions:

- S: Sign the message, which incurs a cost c,
- D: Drop the message which incurs no cost.

The action set of each player i is defined as $\mathcal{A}_i = \{\text{S, D}\}$, and the joint action space is $\mathcal{A} = \mathcal{A}_1 \times \cdots \times \mathcal{A}_N$. The action of player i is denoted by $a_i \in \mathcal{A}_i$, and the joint action vector is $\underline{a} = (a_1, a_2, ..., a_N)$.

Utility Function: Let $k = \sum_{i \in \mathcal{P}} \delta(a_i = \text{S})$ denote the number of players who choose to sign and forward. The threshold condition is satisfied if $k \geq t$, where $t \in \mathbb{N}$ is a predefined protocol parameter. When the threshold is met, the threshold signature is successfully constructed and submitted on-chain. In this case, the oracles are rewarded with a reward r. We define the instantaneous utility $u_i(\underline{a})$ of player i as follows:

$$u_i(\underline{a}) = \begin{cases} r - c & \text{if } a_i = \text{S and } k \geq t, \\ -c & \text{if } a_i = \text{S and } k < t, \\ r & \text{if } a_i = \text{D and } k \geq t, \\ 0 & \text{if } a_i = \text{D and } k < t. \end{cases} \tag{1}$$

C. Players' Strategies and Equilibrium

In this part, we analyse the player's strategies for the one-shot game and derive the Nash Equilibira. To do so, we first recall the definition of the Nash equilibrium in Definition 1.

Definition 1 (Nash Equilibrium, [13]). *Let $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \cdots \times \mathcal{S}_N$ and let $u_i : \mathcal{S} \to \mathbb{R}, i \in \mathcal{N}$. The vector $\underline{s}^*$ is a Nash Equilibrium if:*

$$\forall i \in \mathcal{N},\ \forall s_i \in \mathcal{S}_i, u_i(s_i^*, \underline{s}_{-i}^*) \geq u_i(s_i, \underline{s}_{-i}^*),$$

where $\underline{s}_{-i}^* = (s_1^*, ..., s_{i-1}^*, s_{i+1}^*, s_N^*)$.

Our game closely resembles the public goods game studied in [14], where a discrete public good is produced only if enough players choose to contribute a fixed amount. The authors analyze both settings where contributions are refunded if the good is not provided and where they are not. They show that, in pure strategies, all equilibria without a refund still exist if a refund is provided. In the mixed-strategy setting, allowing refunds increases the expected number of contributors.

Our game corresponds to the special case where each oracle contributes exactly one unit by signing or not, and the public good is the reward when the threshold signature is valid. Since the action is to sign or not, rather than contributing a variable amount, the notion of a refund is not applicable in our context.

We begin by analyzing the outcome of the game in the symmetric case, where all players adopt the same pure strategy. As stated in Proposition 1 of [14], when $t > 1$, there are exactly $\binom{N}{t}$ pure-strategy equilibria with exactly t signers, along with one additional equilibrium in which no player signs. This result is intuitive: if we assume there are t designated oracles who choose to sign, any additional oracle has no incentive to sign as the threshold will be met regardless, and they would still receive the reward. Conversely, if one of the t signers deviates and chooses not to sign, the threshold would no longer be satisfied, and their reward would drop to zero. Thus, exactly t signers form a pure Nash equilibrium, with $\binom{N}{t}$ possible combinations. Moreover, the strategy where no one signs is also a pure-strategy equilibrium: if a single oracle unilaterally decides to sign, the threshold is still not met, and they receive no reward. Hence, there is no incentive to deviate.

Next, we analyze the symmetric mixed-strategy equilibrium of our game. In [15], the authors derive a closed-form expression for the symmetric mixed-strategy equilibrium in a simplified version of a threshold public goods game, under the assumption that exactly t units must be contributed for the public good to be provided. However, this formulation does not directly apply to our setting, since more than t oracles can sign the message and the threshold signature remains valid. Therefore, we proceed with a discussion of the symmetric mixed-strategy equilibrium in our specific context.

Let q be the probability that an oracle chooses to sign the data feed, and $1-q$ the probability of dropping it. Since we are analyzing the symmetric case, all oracles use the same mixed strategy. At equilibrium, no oracle should have an incentive to deviate unilaterally. Therefore, the expected utility of signing, denoted by $U(S, q)$, must equal the expected utility of not signing, denoted by $U(D, q)$.

Let X be the random variable representing the total number of oracles that choose to sign. Since each oracle signs independently with probability q, X follows a binomial distribution with parameters N and q. The probability that the threshold is met (i.e. the message is signed by at least t oracles) is:

$$P(X \geq t) = \sum_{k=t}^{N} \binom{N}{k} q^k (1-q)^{N-k}. \tag{2}$$

Given this, the expected utility of signing is:

$$U(S, q) = r \cdot \sum_{k=t-1}^{N-1} \binom{N-1}{k} q^k (1-q)^{N-1-k} - c, \tag{3}$$

and the expected utility of dropping is:

$$U(D, q) = r \cdot \sum_{k=t}^{N-1} \binom{N-1}{k} q^k (1-q)^{N-1-k} \tag{4}$$

At equilibrium, these two expressions must be equal. Solving the equality $U(S, q) = U(D, q)$ yields the equation:

$$\binom{N-1}{t-1} q^{t-1} (1-q)^{N-t} = \frac{c}{r}. \tag{5}$$

Since solving Equation (5) analytically is untractable for large values of N, we proceed with a numerical evaluation of the mixed-strategy equilibrium in the next section.

IV. Numerical Evaluation

In this section, we numerically evaluate the mixed-strategy equilibria of the proposed game model. To do so, we vary key parameters that influence oracle behavior: the total number of nodes N, the signing threshold t, and the cost-to-reward ratio c/r. For the cost to reward ration, we fix the reward to a unit $r = 1$ and vary c. For each configuration, we compute the expected utilities of signing and dropping the message as functions of the mixed strategy q and identify the symmetric mixed-strategy equilibria. In particular, we investigate: (i) the effect of increasing the number of oracles while maintaining a two-thirds threshold; (ii) the impact of raising the signing cost c; (iii) how the threshold t affects equilibrium strategies for fixed N and c; and (iv) the resulting probability of success at equilibrium for different thresholds.

We begin with the case of $N = 10$ oracles and a threshold set to two-thirds of the total, i.e., $t = \lceil \frac{2N}{3} \rceil$, which reflects a Byzantine fault tolerance requirement. As illustrated in Figure 2, two mixed-strategy equilibria emerge at approximately $q_1 = 0.432$ and $q_2 = 0.855$. However, since the expected utility at q_2 is higher than at q_1, it is reasonable to assume that rational oracles would stabilize around the equilibrium q_2.

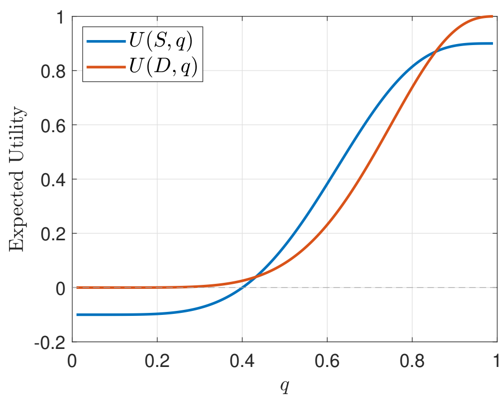

Fig. 2: Expected utilities $U(S,q)$ and $U(D,q)$ as functions of the mixed strategy q, for $N = 10$, $t = \lceil \frac{2N}{3} \rceil$, $r = 1$, and $c = 0.1$.

We then analyze how system parameters, specifically the number of players N, the threshold t, and the cost-to-reward ratio c/r influence the set of equilibria and the corresponding expected utilities. We start by studying the impact of the number of players. Here, we fix the threshold to $t = \lceil \frac{2N}{3} \rceil$ and set the cost c to 0.1, with reward $r = 1$. The results are presented in Figure 3. The figure shows how the number of players N affects the values of the mixed-strategy equilibria. For all evaluated values of N, there are two intersection points corresponding to symmetric mixed-strategy Nash equilibria. When $N = 3$, the lower equilibrium is close to zero, and the upper one is at $q_2 = 0.958$. For $N = 5$, the equilibria are at $q_1 = 0.33$ and $q_2 = 0.97$. At $N = 10$, they shift to $q_1 = 0.43$ and $q_2 = 0.85$. For $N = 25$, the values move inward to $q_1 = 0.56$ and $q_2 = 0.76$. As N increases, the value of q_1 increases while q_2 decreases starting from $N \geq 5$.

Next, we evaluate the impact of increasing the signing cost c on the equilibria and the expected utilities. The results are illustrated in Figure 4. First, we observe that for $c = 0.5$ and $c = 0.7$, no symmetric mixed-strategy equilibrium exists. Additionally, as the cost increases, the expected utility of signing decreases across all values of q, while the utility of dropping remains unchanged. In subplots (a) and (b), corresponding to $c = 0.1$ and $c = 0.2$, two equilibria are observed where the utility curves intersect. At $c = 0.1$, the equilibria occur at $q_1 = 0.43$ and $q_2 = 0.85$, while at $c = 0.2$, they shift to $q_1 = 0.53$ and $q_2 = 0.78$. This shows that the interval between equilibria narrows as the cost increases. In subplot (d), with $c = 0.7$, the utility of signing remains negative for nearly all values of q. Further analysis shows that even at a lower cost of $c = 0.275$, the utility curves do not intersect. These results indicate that higher costs can eliminate the existence of mixed-strategy equilibria. From a system design perspective, this implies that oracles must be sufficiently rewarded for mixed equilibria to emerge, or an explicit coordination mechanism must be established to ensure a coordinated pure-strategy equilibrium.

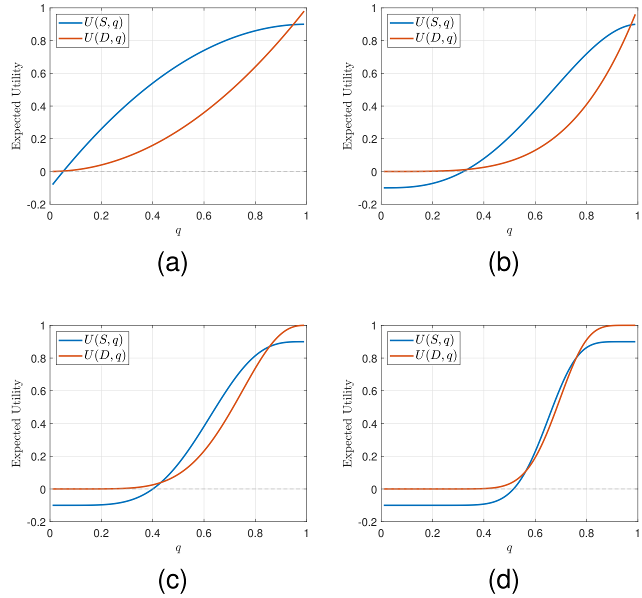

Fig. 3: Expected utilities $U(S,q)$ and $U(D,q)$ as functions of mixed strategy q, and $N = 3, 5, 10$, and 25, $t = \lceil \frac{2N}{3} \rceil$, $r = 1$ and $c = 0.1$. (a) $N = 3$. (b) $N = 5$. (c) $N = 10$. (d) $N = 25$.

Figure 5 shows the effect of varying the threshold t on the mixed-strategy equilibria, with fixed parameters $N = 10$, $r = 1$, and $c = 0.1$. As t increases, the equilibrium points shift to higher values of q. In subplot (a), with $t = 2$, two equilibria are observed at $q_1 = 0.01$ and $q_2 = 0.35$. In subplot (b), for $t = 5$, the equilibria shift to $q_1 = 0.23$ and $q_2 = 0.65$. For $t = 7$ in subplot (c), the values move further to $q_1 = 0.43$ and $q_2 = 0.85$. Finally, in subplot (d), where $t = 10$, only a single equilibrium remains at $q = 0.76$. However, the expected utility at this point is zero. These results indicate that higher thresholds require higher expected participation.

Next, we evaluate the success probability at equilibrium, defined as the probability that at least t oracles sign the

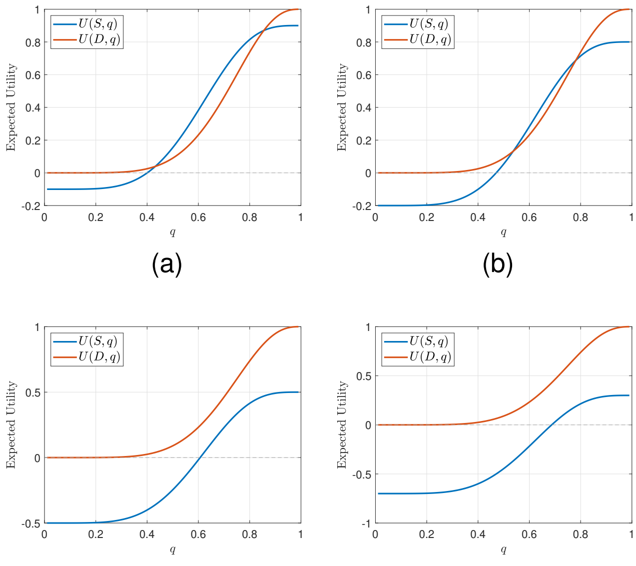

Fig. 4: Expected utilities $U(S,q)$ and $U(D,q)$ as functions of mixed strategy q, and $N = 10$, $t = \lceil \frac{2N}{3} \rceil$, $r = 1$ and $c = 0.1, 0.2, 0.5$ and $c = 0.7$. (a) $c = 0.1$. (b) $c = 0.2$. (c) $c = 0.5$. (d) $c = 0.7$.

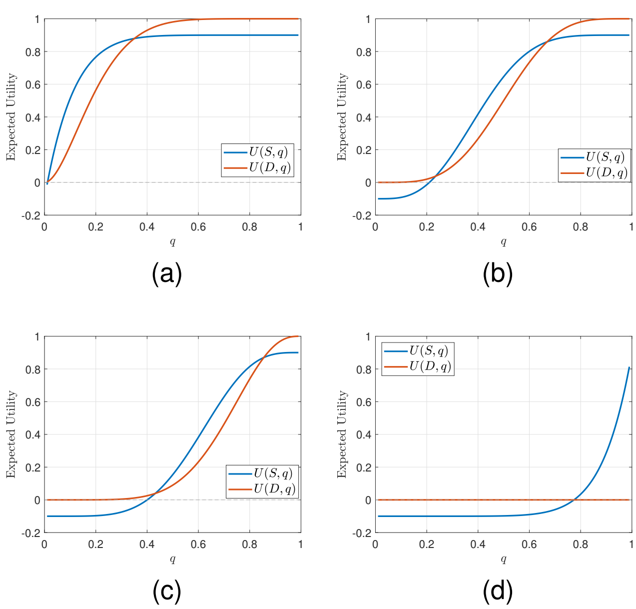

Fig. 5: Expected utilities $U(S,q)$ and $U(D,q)$ as functions of mixed strategy q, and $N = 10$, $r = 1$, $c = 0.1$ and $t = 2, 5, 7$ and $t = 10$. (a) $t = 2$. (b) $t = 5$. (c) $t = 7$. (d) $t = 10$.

message, i.e., $P(X \geq t)$. This is computed using the higher-payoff equilibrium strategy q_2, for a fixed number of oracles $N = 10$, and parameters $r = 1$ and $c = 0.1$, while varying the threshold t from 1 to 9. The results are shown in Figure 6. We observe that the success probability initially decreases as t increases from 1 to 4, reaching a minimum around $t = 4$. Beyond this point, it increases steadily, exceeding 0.95 at $t = 7$. We recall previous findings in Figure 5, which indicate that the equilibrium signing probability q_2 increases with the threshold t. Hence, the curve can be explained as follows: when the threshold is low, even a low signing probability is sufficient to meet the threshold, resulting in a high success probability. As t increases, greater participation among oracles is required, making success less likely unless their participation probability also increases.

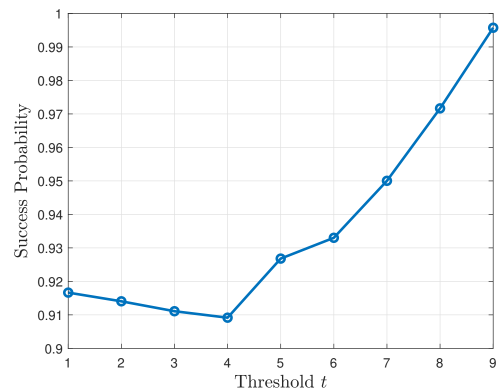

Fig. 6: Probability of success at equilibrium as a function of threshold t for $N = 10$, $r = 1$, $c = 0.1$ and $t = 1, 2, ..., 9$.

In summary, the numerical evaluation highlights how system parameters influence oracle behavior under the assumption of symmetric mixed strategies. The results show that for certain parameter configurations, two mixed equilibria can exist, and as the number of nodes increases, the gap between these equilibrium values narrows. Additionally, higher signing probabilities are associated with better expected payoffs for oracles. In contrast, increasing the signing cost c lowers the utility of contributing and can eventually eliminate mixed-strategy equilibria when the cost becomes too high. Higher threshold values of t shift the equilibria toward higher values of q, requiring stronger belief in others' participation for an individual to participate. These findings suggest that, to ensure oracle participation, the system must either maintain a favorable cost-reward ratio or implement a consensus mechanism that enables a pure strategy equilibrium.

V. Conclusion

We introduced a game-theoretic model for oracle participation in decentralized threshold signature schemes. By modeling the interaction as a discrete public goods game, we analyzed both pure and symmetric mixed-strategy Nash equilibria. Our analysis showed that multiple symmetric mixed-strategy equilibria may emerge depending on key system parameters. Numerical results indicate that the existence of such equilibria is sensitive to the cost-to-reward ratio, while the equilibrium value itself is influenced by the total number of oracles and the threshold size. Notably, even a cost as low as 27.5% of the reward can eliminate symmetric mixed equilibria. In this case, designing a coordination mechanism for oracles to reach pure-strategy equilibria could me more efficient from a system-level perspective. These findings offer useful design insights for building incentive-compatible oracle protocols that rely on threshold participation. Future work could investigate the new equilibria that emerge from repeated oracle interactions, as well as the stability of these equilibria.

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