Stochastic Connected *k*-Coverage in Planar Wireless Sensor Networks Using Optimal Hexagonal Tessellation

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***Abstract*—This paper focuses on the problem of connected *k*-coverage in PWSNs, where every point in a planar field of interest (PFoI) is sensed by at least *k* sensors ($k > 1$) simultaneously, while all the underlying sensors of the network are mutually connected either directly or indirectly. In order to solve this problem, we develop a global framework using an irregular hexagonal tessellation, considering stochastic sensing models for the sensors. In our study, we propose an irregular hexagon, denoted by *IrHx*(r_s/n), for tessellating a PFoI and deploying the sensors with the goal to achieve connected *k*-coverage, where $n > 1$ is a natural number and r_s is the radius of the sensing range of the sensor. First, we tessellate a PFoI with adjacent and congruent regular hexagonal tiles. Then, we modify the properties of this regular hexagonal tile so as to construct our irregular hexagonal tile, *IrHx*(r_s/n), and generate a guided tessellation utilizing the existing regular hexagonal tessellation. Second, we compute the optimal value n^* for generating optimal *IrHx*(r_s/n)-based tessellation. Third, we determine the value of stochastic sensing radius r_s^*, and using this *IrHx*(r_s^*/n^*)-based tessellation configuration, we compute the minimum sensor density, which is required for stochastic *k*-coverage in PWSNs. This helps us compute the minimum number of sensors to *k*-cover a PFoI, while accounting for stochastic sensing model. Next, we establish the necessary relationship for ensuring network connectivity in *k*-covered PWSNs. Finally, we propose our sensor scheduling protocol for stochastic *k*-coverage, and substantiate our theoretical analysis with simulation results.**

Keywords*—*Planar wireless sensor networks, connected k-coverage, stochastic sensing, sensor density, irregular hexagon.

I. Introduction

Sensor *scheduling* (or *duty-cycling*) is one of the most important research problems in planar wireless sensor networks (PWSNs), whose primary goal is to ensure reliable sensing coverage (or simply coverage) of a planar field of interest (PFoI). Indeed, sensor scheduling is an important phase in the design and development of PWSNs that tightly depends on their coverage and connectivity properties, which are in turn the fundamental aspects of sensor deployment. In the real world, there are several sensing applications, such as military surveillance; intruder detection and tracking; forest fire monitoring; flood monitoring; and precision agriculture, which require that every point in a PFoI be monitored by at least one sensor or even k sensors at the same time, where $k \geq 1$ is a natural number, known as the *degree of coverage*. Thus, *k*-coverage is a promising solution as it ensures that every point in a PFoI is covered by at least k sensors at the same time, and enables fault-tolerant data collection throughout the network operation. Also, the *k*-coverage process is performed accurately if the data collected by the sensors can be successfully received by the base station (also known as the sink) for further processing and analysis, making the network connectivity between the sensors and base station essential for *k*-coverage to be meaningful. Therefore, in this paper, we intend to solve the connected *k*-coverage problem in PWSNs.

This paper addresses the problem of connected *k*-coverage in PWSNs leveraging irregular hexagonal tessellation-based approaches using stochastic sensing model, by extending our previous work [5], which focused on the problem of deterministic *k*-coverage in PWSNs. Next, we state the stochastic connected *k*-coverage problem, we address in this paper. Then, we highlight our major contributions.

A. Problem Statement

We intend to solve this problem by addressing the following four major inter-related questions:

— ***Question 1:*** In what optimal way can the sensors be placed in a PFoI such that stochastic *k*-coverage of this PFoI is achieved using a minimum number of sensors, where $k \geq 1$?

— ***Question 2:*** What is the minimum sensor density to provide stochastic *k*-coverage of a PFoI using the above-found sensor placement strategy?

— ***Question 3:*** What is the necessary condition for stochastic network connectivity using this sensor placement strategy?

— ***Question 4:*** How should the sensors be selected and scheduled (or duty-cycled) for *k*-coverage using a deployed sensor density which is near the theoretical sensor density so as to extend the network operational lifetime?

B. Contributions

Our major contributions in this paper can be summarized as follows:

— We compute the optimal value n^* of factor n, that is required to generate the optimal irregular hexagonal tessellation [5].

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— We generalize our irregular hexagonal tessellation-based k-coverage theory [5], using a stochastic sensing model by determining the value of stochastic sensing radius r_s^*.

— We propose a centralized stochastic sensor selection protocol to k-cover a PFoI using a minimum number of sensors with the goal of optimal energy consumption.

— We evaluate the performance of our proposed stochastic connected k-coverage protocol using simulations in comparison to another state-of-the-art tessellation-based stochastic protocol [14].

II. Related Work

Ammari [7] pioneered Reuleaux triangle tessellation-based k-coverage theory and proposed the stochastic SCP_k protocol, which was later succeeded by Regular hexagonal tessellation-based k-coverage theory in [14] that proposed a general purpose stochastic protocol RCH_k. Other notable contributions include Abbasi *et al.* [11] method for coverage control in continuous regions using mobile sensors, Sun *et al.* [10] optimization-based node deployment algorithm, Nakka and Ammari [3, 9] cusp square tessellation for connected k-coverage, Hoyingcharoen and Teerapabkajorndet [13] probabilistic framework for predicting coverage and connectivity levels, and Chenait *et al.* [12] sector-based redundancy algorithms (SRA-Per and SRA-SP) that identify unnecessary sensors by partitioning sensing ranges into predefined sectors. Qiu *et al.* [8] proposed the distributed Voronoi-based cooperation (DVOC) scheme using k-order local Voronoi diagrams and Delaunay triangles to address coverage voids from sensor movement.

III. Preliminaries and Models

In this section, we introduce the terminology and describe the models, such as sensing model, network model, and energy model, which are used in our proposed solution to the problem.

A. Terminology

Definition 1 (*Sensing range*): The *sensing range* of a sensor s, denoted by SA, is the area around the sensor s, where every event in SA is detected by s.

Definition 2 (*Communication range*): The *communication range* of a sensor s, denoted by CA, is the area around the sensor s, where s is able to communicate with any other sensor in CA

Definition 3 (*Planar sensor density*): The *sensor density* is the number of sensors per unit area that is required to k-cover an area.

Definition 4 (*Tessellation*): The *tessellation* of a PFoI is the process of overlaying the entire PFoI with adjacent and non-overlapping copies of a geometric shape without causing any gaps.

Definition 5 (*Tile*): The *tile* is a convex polygonal shape that is used for generating a tessellation of a PFoI.

B. Sensing Model

We consider stochastic sensing model to develop the theory for solving the problem of connected k-coverage, where r_s and r_c are the radii of sensing and communication ranges respectively. The coverage of point P by the sensor s is defined as the coverage function $Cov(P, s)$. Taking signal attenuation and noisy sensor readings into consideration, we used stochastic sensing model which is more realistic that considers $Cov(P, s)$ as probability of detection $p(P, s)$, as in (2).

$$p(P,s) = \begin{cases} e^{-\beta\delta(P,s)^{\alpha}} & if\ \delta(P,s) \le r_s \\ 0 & otherwise \end{cases} \tag{1}$$

where β represents the physical characteristics of the sensor's sensing unit, and $\alpha \in [2, 4]$ is the path-loss exponent.

C. Network Model

Assumption 1 (*Sensor deployment*): The sensors are randomly and densely deployed in a PFoI.

Assumption 2 (*Location Awareness*): The sensors and base station (or the sink) are aware of their own locations through a global positioning system (GPS) or a localization technique [1].

Assumption 3 (*Homogeneous sensors*): All the sensors have the same characteristics, including their initial energy, sensing and communication capabilities, and movement speed.

Assumption 4 (*Sensor Mobility*): All the sensors are mobile and can freely move to specific locations in a PFoI.

D. Energy Model

We use the energy model proposed by Heizelman *et al.* [2], for computing the energy consumed due to data transmission and data reception as defined below, respectively:

$$E_t(d) = b \times (\varepsilon d^{\alpha} + E_e) \tag{2}$$

$$E_r = b \times E_e \tag{3}$$

where $E_t(d)$ is the energy consumed by sensor s while transmitting a message of b bits over a distance d, E_r is the energy consumed by sensor s while receiving a message of b bits, E_e is the electronical energy, ε is the transmitter amplifier in the free-space (ε_{fs}) or multi-path (ε_{mp}) model, and $\alpha \in [2, 4]$ is the path-loss exponent.

In addition, we considered the energy model provided by Ye *et al.* [4] for computing the energy consumed by the sensors for phenomenon sensing purposes, where a sensor consumes 0.012 J in idle mode (E_{idle}), 0.0003 J in sleep mode (E_{sleep}) and the energy consumed by sensor mobility per distance moved (E_{move}) is selected randomly from interval [0.008, 0.012] J/m [6], such that the total energy consumption for sensor mobility E_m is computed using distance moved (d_{move}) as:

$$E_m = d_{move} \times E_{move} \tag{4}$$

IV. Hexagonal Tessellation-based k-Coverage

In this section, we investigate the problem of connected k-coverage of a PFoI in PWSNs using hexagonal tiles.

A. Irregular Hexagon-based k-Coverage Theory

We tessellate a PFoI using regular hexagons of side length r_s, where r_s is the sensing range of the sensors, as shown in

Figure 1. We then modify the regular hexagon tessellation by updating its side length to $r_s/2$. Now we consider a diamond area in the tessellation and construct the largest common sensing area by drawing four circles centered at the vertices of a diamond area with radius r_s. This largest common sensing area creates our irregular hexagon *IrHx*($r_s/2$) shown in Figure 2, where k sensors should be placed in the diamond area for attaining k-coverage of the irregular hexagon *IrHx*($r_s/2$). Similarly, we can generate the irregular hexagons *IrHx*($r_s/3$), *IrHx*($r_s/4$) and *IrHx*($r_s/5$), by modifying the side length of regular hexagon tessellation to $r_s/3$, $r_s/4$, and $r_s/5$, respectively, as shown in Figure 3. It is clear that the area of these shapes increases in the order *IrHx*($r_s/2$) < *IrHx*($r_s/3$) < *IrHx*($r_s/4$) < *IrHx*($r_s/5$) as we increase the value of n from 2 to 5.

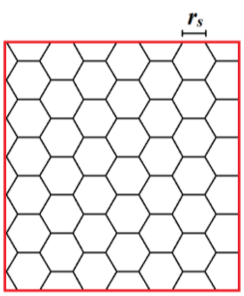

Fig. 1. Regular hexagon tessellation for FoI.

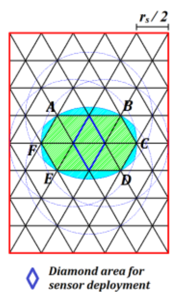

Fig. 2. Construction of Irregular Hexagon *IrHx*($r_s/2$).

Generic Irregular Hexagon *IrHx*(r_s/n): Let us consider a more generic side length r_s/n for the regular hexagonal tessellation to create our generic irregular hexagon *IrHx*(r_s/n). As noted from previous cases (n = 2, 3, 4, 5), our *IrHx*(r_s/n) has a certain structure in terms of the lengths of six sides, *i.e.*, $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$ and $\overline{FA}$ and the number of rings of equilateral triangles comprising it. Table I demonstrates the structure of the irregular hexagons *IrHx*($r_s/2$), *IrHx*($r_s/3$), *IrHx*($r_s/4$) and *IrHx*($r_s/5$). Based on these results, the structure of *IrHx*(r_s/n) is illustrated in Table II. Next, we will develop our computational geometry-based approach, which ensures connected k-coverage of a PFoI using our proposed irregular hexagon, *IrHx*(r_s/n).

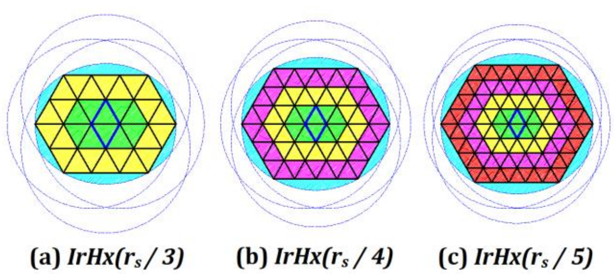

Fig. 3. Different irregular hexagon configurations.

Lemma 1 below, computes the number of equilateral triangles for any ring l of our generic irregular hexagon *IrHx*(r_s/n).

TABLE I. STRUCTURE OF *IRHX*(R_s/N) FOR DIFFERENT VALUES OF *N*.

n	$\overline{AB}$	$\overline{BC}$	$\overline{CD}$	$\overline{DE}$	$\overline{EF}$	$\overline{FA}$	*#Rings*
2	r_s	$r_s/2$	$r_s/2$	r_s	$r_s/2$	$r_s/2$	1
3	r_s	$2r_s/3$	$2r_s/3$	r_s	$2r_s/3$	$2r_s/3$	2
4	r_s	$3r_s/4$	$3r_s/4$	r_s	$3r_s/4$	$3r_s/4$	3
5	r_s	$4r_s/5$	$4r_s/5$	r_s	$4r_s/5$	$4r_s/5$	4

TABLE II. GENERIC STRUCTURE OF *IRHX*(R_s/N).

$\overline{AB}$	$\overline{BC}$	$\overline{CD}$	$\overline{DE}$	$\overline{EF}$	$\overline{FA}$	*#Rings*
r_s	$\frac{(n-1)r_s}{n}$	$\frac{(n-1)r_s}{n}$	r_s	$\frac{(n-1)r_s}{n}$	$\frac{(n-1)r_s}{n}$	$n-1$

Lemma 1 (*Number of triangles per ring*) [5]: In our generic irregular hexagon *IrHx*(r_s/n), the number of equilateral triangles in ring l, denoted by N_l, can be computed as:

$$N_l = 12l - 2$$

Leveraging the results of Lemma 1, Lemma 2 below computes the total number of equilateral triangles, which constitute our generic irregular hexagon, *IrHx*(r_s/n).

Lemma 2 (*Total number of triangles*) [5]: The total number of equilateral triangles N, that comprise our *IrHx*(r_s/n), is given by:

$$N = 2(n-1)(3n-1)$$

Theorem 1 (*Planar Sensor Density*): The planar sensor density $\lambda(k, r_s, n)$, which is required to k-cover a field of interest, is computed as follows:

$$\lambda(k, r_s, n) = \frac{2n^2 k}{\sqrt{3}(n-1)(3n-1)r_s^2}$$

where r_s is the sensing radius of sensor, factor $n > 1$, and degree of coverage $k \geq 1$.

Proof: The planar sensor density of an irregular hexagonal tile is the number of sensors deployed per unit area of the tile. Hence, given that k sensors need to be placed within the inner diamond area to k-cover an area whose size is A_{IrHx}, this planar sensor density, denoted by $\lambda(k, r_s, n)$, is given by:

$$\lambda(k, r_s, n) = \frac{k}{A_{IrHx}} = \frac{k}{\frac{\sqrt{3}}{2}(n-1)(3n-1)\left(\frac{r_s}{n}\right)^2} = \frac{2n^2 k}{\sqrt{3}(n-1)(3n-1)r_s^2}$$

Lemma 3 below establishes the necessary relationship between the radii of the sensing and communication ranges of the sensors for maintaining network connectivity of PWSNs. This helps ensure connected k-coverage during the network operation of PWSNs using our generic irregular hexagon, *IrHx*(r_s/n).

Lemma 3 (*Network Connectivity*) [5]: An Irregular hexagonal tessellation-based k-coverage configuration is said to be connected if the radii of sensing and communication ranges of the sensors, r_s and r_c, respectively, obey the inequality below, where factor $n > 1$:

$$r_c \geq \frac{\sqrt{3n^2+1}}{n} r_s$$

B. Optimal value of 'n'

In order to maximize PWSN operational lifetime, we must achieve k-coverage using the optimal number of sensors, which requires minimizing planar sensor density by maximizing the k-covered area of our generic irregular hexagon $IrHx(r_s/n)$. However, maximizing k-covered area reduces the inner diamond sensor placement area, potentially causing interference issues. Therefore, we must standardize $IrHx(r_s/n)$ to achieve an optimal trade-off between maximum k-covered area and maximum sensor placement area.

Theorem 2 below determines the optimal value of n that achieves this trade-off of maximum k-covered area and maximum sensor placement area.

Theorem 2 (*Optimal value of n*): The optimal value n^* of factor n that achieves the trade-off of maximum k-covered area and maximum sensor placement area, is given by:

$$n^* = 0.5 + 6\frac{w_1}{w_2}$$

where $0 < w_1, w_2 < 1$, and $w_1 + w_2 = 1$

Proof: We formulate this trade-off problem as a multi-objective function and solve it using weighted scale-uniform-unit sum approach for determining the optimal value n^* of factor n. Let $\mathcal{F}(n) = w_1c_1|A_{InDi}| + w_2c_2|A_{IrHx}|$, be our multi-objective function which we want to maximize. The values of weighted coefficients w_1, w_2 is $0 < w_1, w_2 < 1$, and $w_1 + w_2 = 1$, reflects the importance of maximizing sensor placement area A_{InDi} and k-covered area A_{IrHx} respectively. Therefore, our unconstrained optimization problem can be formulated as,

$$\text{Maximize } \mathcal{F}(n) = w_1c_1|A_{InDi}| + w_2c_2|A_{IrHx}|, \quad n > 1$$

where,

$$c_1 = \frac{\omega}{|A_{InDi}|_{max}}$$

$$c_2 = \frac{\omega}{|A_{IrHx}|_{max}}$$

and $\omega = \max\{|A_{InDi}|_{max}, |A_{IrHx}|_{max}\}$

with $|A_{InDi}|_{max}$ and $|A_{IrHx}|_{max}$ being maximum sensor placement area and maximum k-covered area respectively. We have,

$$|A_{InDi}|_{max} = \text{Sensor placement area of } IrHx(r_s/2) = \frac{\sqrt{3}}{8}r_s^2$$

$$|A_{IrHx}|_{max} = \lim_{n\to\infty}\frac{\sqrt{3}}{2}(n-1)(3n-1)\left(\frac{r_s}{n}\right)^2 = \frac{3\sqrt{3}}{2}r_s^2$$

Therefore, we get,

$$\omega = \max\{|A_{InDi}|_{max}, |A_{IrHx}|_{max}\} = \frac{3\sqrt{3}}{2}r_s^2$$

$$c_1 = 12 \text{ and } c_1 = 1$$

Therefore, the function $\mathcal{F}(n)$ can be written as follows:

$$\mathcal{F}(n) = \frac{\sqrt{3}}{2}r_s^2\left[\frac{12w_1}{n^2} + \frac{w_2(n-1)(3n-1)}{n^2}\right]$$

We want to study our multi-objective function $\mathcal{F}(n)$. Let n^* be the solution to our stated unconstrained optimization problem. We have:

$$\frac{\partial\mathcal{F}(n)}{\partial n} = 0 \Rightarrow n^* = 0.5 + 6\frac{w_1}{w_2}$$

In order to verify that n^* corresponds to the maximum value of $\mathcal{F}(n)$, we have,

$$\frac{\partial^2\mathcal{F}(n)}{\partial n^2} > 0$$

whenever the following inequality holds,

$$n^* < \hat{n} = 0.75 + 9\frac{w_1}{w_2}$$

Given that $n^* < \hat{n}$, it is clear that the n^* corresponds to the maximum value of $\mathcal{F}(n)$. Furthermore, varying the weights w_1 and w_2 from 0 to 1, where $w_1 + w_2 = 1$, would generate the corresponding maximum values of $\mathcal{F}(n^*)$.

C. Stochastic Connected k-Coverage

In this section, we exploit the results of section IV.A for characterizing the stochastic k-coverage.

Definition 6 (*Stochastic k-coverage*): A point P in a PFoI is said to be probabilistically k-covered if the probability of detection of an event occurring at P by at least k sensors is at least equal to certain threshold probability p_{th}, where $0 < p_{th} < 1$.

Theorem 3 below computes the minimum probability of detection p_{min} required for k-coverage.

Theorem 3 (*Minimum k-coverage probability*): The minimum required probability of detection for k-coverage using our stochastic sensing model is given by

$$p_{min} = 1 - \left(1 - e^{-\beta r_s^{\alpha}}\right)^k$$

where r_s is the sensing radius of sensor.

Proof: In order to compute p_{min}, we identify the specific point in a PFoI that is the least possibly k-covered. Based on our sensor placement strategy, we deployed k sensors in the inner diamond area of our $IrHx(r_s/n)$. It is clear that point A is the least possibly k-covered if all the k sensors are deployed on the lower right side of the inner diamond area (from Figure 2). Then, the highest distance between point A and k sensors will be exactly r_s. Therefore, the minimum required probability of detection p_{min} for the least possible k-covered point A, using our stochastic sensing model, is given by,

$$p_{min} = 1 - \prod_{i=1}^{k}(1 - p(P, s_i)) = 1 - \left(1 - e^{-\beta r_s^{\alpha}}\right)^k$$

In order to solve the stochastic k-coverage problem, we have to select a minimum subset S_{min} of sensors, where $S_{min} \subseteq S$, such that every point in a PFoI is probabilistically k-covered by at least k sensors with a probability of detection that is at least equal to p_{th}. This allows us to compute the minimum stochastic sensing radius r_s^* that allows us to achieve stochastic k-coverage of a PFoI with probability no less than p_{th}. Lemma 4 below computes the stochastic sensing radius r_s^*.

Lemma 4 (*Stochastic Sensing Radius*): The minimum stochastic sensing radius r_s^* that is required for achieving stochastic k-coverage of a PFoI is computed as follows:

$$r_s^* = \left(-\frac{1}{\beta}\, ln\left(1-(1-p_{th})^{1/k}\right)\right)^{1/\alpha}$$

where β represents the physical characteristics of the sensor's sensing unit, $\alpha \in [2, 4]$, and $k \geq 1$.

Proof: From Definition 6 and Theorem 2, we have,

$$p_{min} \geq p_{th}$$

$$\Rightarrow 1-\left(1-e^{-\beta r_s^{\alpha}}\right)^k \geq p_{th}$$

$$\Rightarrow r_s \geq \left(-\frac{1}{\beta}\, ln\left(1-(1-p_{th})^{1/k}\right)\right)^{1/\alpha}$$

Therefore, minimum stochastic sensing radius r_s^* is given by,

$$r_s^* = \left(-\frac{1}{\beta}\, ln\left(1-(1-p_{th})^{1/k}\right)\right)^{1/\alpha}$$

We leverage this minimum stochastic sensing radius r_s^* as a key parameter for solving the stochastic k-coverage problem based on our stochastic sensing model. Incorporating the results from Theorem 2 and Lemma 4, we reformulate Theorem 1 and Lemma 3 as Theorem 4 and Lemma 5 respectively to account for stochastic behavior.

Theorem 4 (*Stochastic Sensor Density*): The stochastic sensor density $\lambda^*(k, r_s^*, n^*)$, which is required to k-cover a PFoI, is computed as follows:

$$\lambda^*(k, r_s^*, n^*) = \frac{2(n^*)^2 k}{\sqrt{3}(n^*-1)(3n^*-1)(r_s^*)^2}$$

where r_s^* is the minimum stochastic sensing radius, degree of coverage $k \geq 1$ and factor $n > 1$.

Lemma 5 (*Stochastic Network Connectivity*): An Irregular hexagonal tessellation-based k-coverage configuration is said to be connected if the stochastic communication radius r_c^* and stochastic sensing radius r_s^*, satisfy the inequality below, where factor $n > 1$:

$$r_c^* \geq \frac{\sqrt{3(n^*)^2+1}}{n^*} r_s^*$$

D. Stochastic k-Coverage Protocol: St-k-InDi

By taking all these mathematical relations from previous section into account, accommodating the stochastic parameters and constraints, we propose our ***St***ochastic ***k***-coverage protocol using ***In***ner ***Di***amonds (St-k-InDi). Our stochastic protocol has two phases, where phase-1 generates the stochastic irregular hexagonal tessellation and phase-2 is responsible for the selection and scheduling of sensors to minimize the sensors' energy consumption during the k-coverage process, thus, maximizing the network operational lifetime. Figure 4 illustrates the state transition of sensors for duty-cycling followed in phase-2 of our St-k-InDi protocol.

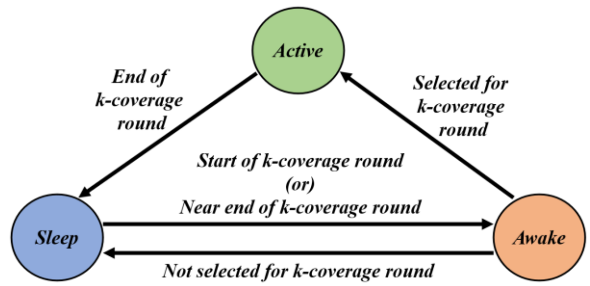

Fig. 4. Sensor's state transition diagram for duty-cycling.

V. Performance Evaluation

This section presents simulation results of our stochastic connected k-coverage protocol St-k-InDi using an open-source high-level simulator built with C and Python. Due to space limitation, we provide only the simulation results presented in Figures 5-7.

A. Simulation Results of St-k-InDi

Figure 5 shows the plot of the stochastic sensor density λ* (Theorem 4) for α = 2 (Figure 5a) and α = 4 (Figure 5b), and w_1 = w_2 = 0.5, while varying k. For constant α, we observe that λ* increases with an increase in k. Also, it is evident that for higher threshold probability p_{th} with constant α, higher λ* is required to achieve the same degree of coverage k. In case of α = 2, there is a slight deviation from the expected behavior of λ* versus k, while for α = 4, the behavior of λ* versus k is as expected, i.e., λ* is directly proportional to the degree of coverage, k.

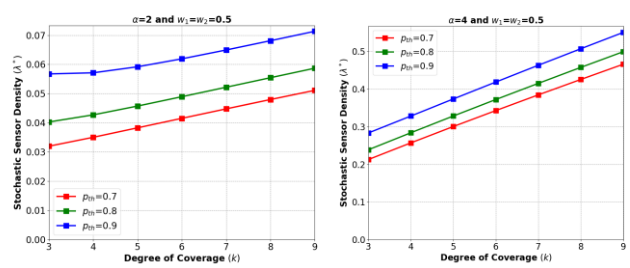

Fig. 5. Stochastic planar sensor density λ^* versus Degree of coverage k for (a) $\alpha = 2$ and (b) $\alpha = 4$

B. Comparison of St-k-InDi with RCH$_k$

The plots in Figure 6 shows the difference between St-k-InDi and RCH$_k$ protocols in terms of number of active sensors n_a for stochastic k-coverage. Figure 6(a) shows that n_a stays constant for any number of deployed sensors n_d and Figure 6(b) indicates that the required number of active sensors n_a to ensure 3-coverage increases with the physical characteristics of sensor's sensing unit β. Clearly, both plots indicate the usage of a lesser number of active sensors n_a for our St-k-InDi protocol compared to RCH$_k$ [14].

Figure 7(a) plots the degree of coverage k versus the required number of active sensors, n_a for stochastic k-coverage for St-k-InDi and RCH_k [14] protocols, with degree of coverage $k = 3$, path-loss exponent $\alpha = 2$ and the least threshold probability of $p_{th} = 0.7$. Figure 7(b) compares the performance of St-k-InDi with RCH_k [14], in terms of their energy consumption and operational network lifetime. As discussed in the previous sections, by minimizing the energy consumption per k-coverage round of operation, we can extend the operational lifetime of the PWSN monitoring a PFoI. It is clear that our protocol St-k-InDi has a higher operational network lifetime compared to RCH_k [14], thus, achieving optimal energy consumption for similar experimental conditions.

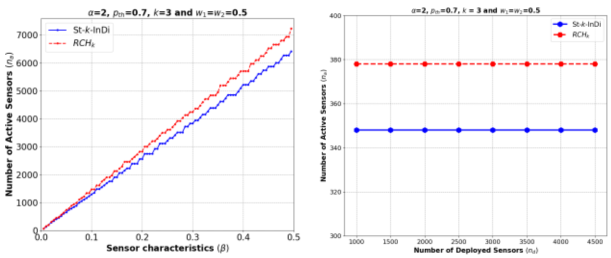

Fig. 6. Number of active sensors n_a versus (a) Sensor characteristics β and (b) Number of deployed sensors n_d

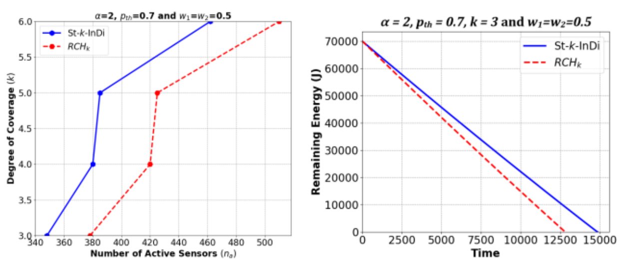

Fig. 7. Comparing St-k-InDi and RCH_k: (a) Degree of coverage k versus Number of active sensors n_a and (b) Remaining energy versus Time

VI. Conclusion

In this paper, we investigated the stochastic connected k-coverage problem in PWSNs using an irregular hexagonal tessellation-based approach. We computed the optimal slicing factor n^*, stochastic sensing radius $r_s{}^*$, and determined the required stochastic sensor density for k-coverage while establishing necessary connectivity relationships, for extending our irregular hexagonal tessellation-based k-coverage [5]. Based on these mathematical properties, we proposed the St-k-InDi protocol, which outperforms the existing RCH_k protocol [14] in terms of active sensors usage, sensor density, energy consumption, and network lifetime under worst-case threshold sensing probability conditions.

Our future work is three-fold: (1) extending our irregular hexagonal tessellation theory to heterogeneous sensors with different characteristics [15], (2) extending our approach to stochastic connected k-coverage in 3D WSNs, and (3) implementing our protocols on a real-world sensor testbed [16].

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References

[1] N. Bulusu, J. Heidemann, and D. Estrin, "GPS-less low-cost outdoor localization for very small devices," *IEEE Personal Communications*, 7(5), 2000.

[2] W. B. Heinzelman, A. P. Chandrakasan, and H. Balakrishnan, "An application-specific protocol architecture for wireless microsensor networks," *IEEE Trans. Wirel. Comm.*, 1(4), 2002.

[3] K. Nakka and H. M. Ammari, "k-CSqu: Ensuring connected k-coverage using cusp squares of square tessellation," *J. Parallel Distrib. Comput.*, vol. 182, no. 104749, p. 104749, 2023.

[4] F. Ye, G. Zhong, J. Cheng, S. Lu, and L. Zhang, "PEAS: a robust energy conserving protocol for long-lived sensor networks," *Proc. EEE ICDCS*, 2004.

[5] K. Nakka and H. M. Ammari, "An energy-efficient irregular hexagonal tessellation-based approach for connected k-coverage in planar wireless sensor networks," *Ad Hoc Netw.*, vol. 154, no. 103353, p. 103353, 2024.

[6] Y.-C. Wang and Y.-C. Tseng, "Distributed deployment schemes for mobile wireless sensor networks to ensure multilevel coverage," *IEEE Transactions on Parallel and Distributed Systems*, 19(9), pp. 1280–1294, 2008.

[7] H. M. Ammari, "Stochastic *k*-coverage in wireless sensor networks," in *Wireless Algorithms, Systems, and Applications*, pp. 125–134, 2009.

[8] C. Qiu, H. Shen, and K. Chen, "An energy-efficient and distributed cooperation mechanism for *k*-coverage hole detection and healing in WSNs," *Proc. IEEE 12th International Conference on Mobile Ad Hoc and Sensor Systems*, 2015.

[9] K. Nakka and H. M. Ammari, "Square tessellation for stochastic connected k-coverage in planar wireless sensor networks," in *2023 IEEE Symposium on Computers and Communications (ISCC)*, 2023, pp. 864–867.

[10] Z. Sun, C. Li, X. Xing, H. Wang, B. Yan, and X. Li, "*K*-degree coverage algorithm based on optimization nodes deployment in wireless sensor networks," *International Journal on Distributed Sensor Networks*, 13(2), 2017.

[11] F. Abbasi, A. Mesbahi, and J. Mohammadpour Velni, "A new Voronoi-based blanket coverage control method for moving sensor networks," *IEEE Transactions on Control Systems Technology*, 27(1), pp. 409–417, 2019.

[12] M. Chenait, B. Zebbane, and N. Badache, "A new *k*-coverage model to determine redundant sensors in wireless sensor networks," *Proc. International Conference on Smart Communications in Network Technologies* (SaCoNeT), 2018.

[13] P. Hoyingcharoen and W. Teerapabkajorndet, "Expected probabilistic detection and sink connectivity in wireless sensor networks," *IEEE Sensors Journal*, 19(12), pp. 4480–4493, 2019.

[14] H. M. Ammari, "Connected *k*-coverage in two-dimensional wireless sensor networks using hexagonal slicing and area stretching," *Elsevier JPDC*, vol. 153, pp. 89–109, 2021.

[15] K. Nakka and H. M. Ammari, "Hierarchical deployment and square tessellation for connected k-coverage in heterogeneous planar wireless sensor networks," *ACM Trans. Sens. Netw.*, vol. 21, no. 2, pp. 1–27, 2025.

[16] P. Appavoo, E. K. William, M. C. Chan, and M. Mohammad, "Indriya2: A Heterogeneous Wireless Sensor Network (WSN) Testbed," in *Lecture Notes of the Institute for Computer Sciences, Social Informatics and Telecommunications Engineering*, Cham: Springer International Publishing, 2019, pp. 3–19.